Find the radius (and interval) of convergence for

1. Use the function $f(x) = \cos x$ to form the Maclaurin series
$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$
   (a) Find the derivatives

   (b) evaluate the derivatives at zero

   (c) Assemble the series

   (d) Use the ratio test to conclude that the series converges for all x

   (e) What is the upper bound of $\left|f^{(n+1)}(z)\right|$

2. Show that the Maclaurin series for $f(x) = \cos x$ converges to $\cos x$ for all x (Use Th 9.19, and show the remainder is zero by squeezing it between 0 and $\dfrac{|x|^{n+1}}{(n+1)!}$)

3. Find the Maclaurin series for $f(x) = \cos x^2$

   Instead of finding the derivatives, and finding $f'(0), f''(0), \ldots$, consider the known power series for $g(x) \cos x$. since $f(x) = g(x^2)$ we have only to substitute to obtain the power series.

4. Find the Maclaurin series for $f(x) = (1+x)^k$

   (a) Find the derivatives

   (b) evaluate the derivatives at zero

   (c) Assemble the series

   (d) Use the ratio test to the radius and interval of convergence

5. Using the known series for the basic functions find the Maclaurin series for $f(x) = \sqrt[3]{1+x}$

6. Using the known series for the basic functions find the Maclaurin series for $f(x) = \cos(\sqrt{x})$

7. Using the known series for the basic functions find the first three non-zero terms of the Maclaurin series for $f(x) = e^x \arctan x$

8. Using the known series for the basic functions find the first three non-zero terms of the Maclaurin series for $f(x) = \sin^2 x = \dfrac{1 - \cos(2x)}{2}$

9. Use a power series to approximate

$$\int_0^1 e^{-x^2}\,dx$$

with an error of less than 0.01.

(a) Use the power series for e^x, replacing x with $-x^2$

(b) Using the Alternating series Remainder Th, which term is less than 0.01?

(c) So you should add how many terms?

(d) Your estimate is then...

Answers:

4. $R = 1$, (-1,1)

9. the fifth term is $\frac{1}{216} < \frac{1}{100}$, so use the sum of the first 4 terms to estimate 0.74