Calculus AB *Derivatives & Integral Rules*

Properties:

$\frac{d}{dx}(c)=0$ $(c\cdot f)'=c\cdot f'$ $(f\pm g)'=f'\pm g'$

$\frac{d}{dx}\left(\int f(x)dx\right)=f(x)$ $\int c\cdot f(x)dx=c\int f(x)dx$

$\int\left(f(x)\pm g(x)\right)dx=\int f(x)dx\pm\int g(x)\,dx$

Derivative Rules:

Product:

$(f\cdot g)'=f\cdot g'+g\cdot f'$

Quotient:

$\left(\frac{f}{g}\right)'=\frac{g\cdot f'-f\cdot g'}{g^2}$

Chain Rule (Composition of functions):

$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$, *or* $\frac{d}{dx}(f(u))=f'(u)\cdot\frac{du}{dx}$, *or* $\frac{d}{dx}(f(g(x)))=f'(g(x))\cdot g'(x)$

Integration: Use Algebra, Trigonometry, or u-substitution to get the integral into one of the patterns below.

Derivatives & Integrals:

Chain rule	Integral(Antiderivative)
$\frac{d}{dx}(u^n)=n\cdot u^{n-1}\frac{du}{dx}$	$\int u^n du=\frac{u^{n+1}}{n+1}+c$
$\frac{d}{dx}(\sin u)=\cos u\frac{du}{dx}$	$\int\cos u\,du=\sin u+c$
$\frac{d}{dx}(\cos u)=-\sin u\frac{du}{dx}$	$\int\sin u\,du=-\cos u+c$
$\frac{d}{dx}(\tan u)=\sec^2 u\frac{du}{dx}$	$\int\sec^2 u\,du=\tan u+c$
$\frac{d}{dx}(\cot u)=-\csc^2 u\frac{du}{dx}$	$\int\csc^2 u\,du=-\cot u+c$
$\frac{d}{dx}(\sec u)=\sec u\tan u\frac{du}{dx}$	$\int\sec u\tan u\,du=\sec u+c$
$\frac{d}{dx}(\csc u)=-\csc u\cot u\frac{du}{dx}$	$\int\csc u\cot u\,du=-\csc u+c$
$\frac{d}{dx}(e^u)=e^u\frac{du}{dx}$	$\int e^u du=e^u+c$
$\frac{d}{dx}(\ln\lvert u\rvert)=\frac{1}{u}\frac{du}{dx}$	$\int\frac{du}{u}=\ln\lvert u\rvert+c$

$$\frac{d}{dx}\left(\log_a |u|\right)=\frac{1}{u\ln a}\frac{du}{dx}$$

$$\frac{d}{dx}\left(a^u\right)=a^u \ln a\frac{du}{dx}$$

$$\int a^u du=\frac{a^u}{\ln a}+c$$

$$\frac{d}{dx}\left(\sin^{-1} u\right)=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\int\frac{du}{\sqrt{a^2-u^2}}=\sin^{-1}\left(\frac{u}{a}\right)+c$$

$$\frac{d}{dx}\left(\cos^{-1} u\right)=-\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\tan^{-1} u\right)=\frac{1}{1+u^2}\frac{du}{dx}$$

$$\int\frac{du}{a^2+u^2}=\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+c$$

$$\frac{d}{dx}\left(\cot^{-1} u\right)=-\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\sec^{-1} u\right)=\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\int\frac{du}{|u|\sqrt{u^2-a^2}}=\frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right|+c$$

$$\frac{d}{dx}\left(\csc^{-1} u\right)=-\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\int \tan u\, du=\ln|\sec u|+c$$

$$\int \cot u\, du=\ln|\sin u|+c$$

$$\int \sec u\, du=\ln|\sec u+\tan u|+c$$

$$\int \csc u\, du=\ln|\csc u-\cot u|+c$$

Calculus AB often used formulas

1) Calculator in Radian Mode

2) Continuity and Differentiability:

$$f(x)=\begin{cases} g(x), & x<a \\ h(x), & x\ge a \end{cases} \qquad \text{then} \quad f'(x)=\begin{cases} g'(x), & x<a \\ h'(x), & x>a \end{cases}$$

Continuity: $\lim\limits_{x\to a^-} g(x)\overset{?}{=}h(a)\overset{?}{=}\lim\limits_{x\to a^+} h(x)$

Differentiability: Continuity plus $\lim\limits_{x\to a^-} g'(x)\overset{?}{=}\lim\limits_{x\to a^+} h'(x)$

3) $Area=\int_a^b height\,dx,\qquad Volume=\int_a^b Area\;dx$

4) Relative Max/Min of $k(x)\to k'(x)=0$ *or und* (critical points)

Max $\to k'$ *changes from* $+$ *to* $-$

Min $\to k'$ *changes from* $-$ *to* $+$

5) Absolute Max/Min (max/min) of $k(x)$ @ critical points or endpoints

6) Points of inflection of $k(x)\to k''(x)=0$ *or und and* k'' *changes sign*

7) Average value: $f_{ave}=\dfrac{1}{b-a}\int_a^b f(x)dx$

Application of average value: $v_{ave}=\dfrac{1}{b-a}\int_a^b v(t)dt=\dfrac{s(b)-s(a)}{b-a}$

8) Position-Velocity-Acceleration:

$$v(t)=s'(t) \quad\to\quad \int v(t)dt=s(t)+c$$

$$a(t)=v'(t)=s''(t) \quad\to\quad \int a(t)dt=v(t)+c$$

9) Initial condition formula: (x_0,y_0), $y(x)=y_0+\int_{x_0}^{x} y'(t)dt$

Application (t_0,s_0), $s(t)=s_0+\int_{t_0}^{t} v(x)dx$

10) Distance- Displacement:

$$Distance = \int_a^b |v(t)|\,dt \rightarrow area$$

$$Displacement = \int_a^b v(t)\,dt \rightarrow net\ signed\ area$$

11) Fundamental Theorem of Calculus

$$\int_a^b f'(x)dx = f(b) - f(a)$$

$$\frac{d}{dx}\int_a^{f(x)} h(t)dt = h(f(x))\cdot f'(x)$$

12) Given a graph of a function f,

$$and\ g(x) = \int_a^x f(t)dt,\ then$$

$$g'(x) = f(x)\quad make\ sign\ line\ here$$

$$g''(x) = f'(x)\quad make\ sign\ line\ here$$

13) Related rates: $find \frac{dV}{dt},\ given \frac{dr}{dt}\ and\ V = \pi r^2 h$

Find connection between r and h and substitute (in cones look for similar triangles). Differentiate with respect to t. Don't forget the chain rule.

$$V = \pi r^2 h,\ h = 2r$$

Example: $V = \pi r^2(2r) = 2\pi r^3$

$$\frac{dV}{dt} = 2\pi\left(3r^2\frac{dr}{dt}\right)$$

14) Slope fields: Slope field given- $\frac{dy}{dx} = x^2(y-1)$

Solve: find y - separate variables, integrate both sides, only one c is necessary, use initial condition. If 2 branches, use the branch the contains the point given (initial value)

Trigonometry Functions

Reciprocal functions $\sin\theta = \dfrac{1}{\csc\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$

Functions in terms of sin & cos $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\vartheta = \dfrac{\cos\vartheta}{\sin\vartheta}$ $\csc x = \dfrac{1}{\sin x}$

$\sec x = \dfrac{1}{\cos x}$

Negative functions

$\sin(-\theta) = -\sin\theta$ *(odd function)*

$\cos(-\theta) = \cos\theta$ *(even function)*

$\tan(-\theta) = -\tan\theta$ *(odd function)*

Cofunctions: sine – cosine, etc

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta, \quad \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta, \quad \sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta, \quad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ *divide by* $\sin^2\theta \rightarrow 1 + \cot^2\theta = \csc^2\theta$ *divide by* $\cos^2\theta \rightarrow \tan^2\theta + 1 = \sec^2\theta$

Cosine: Sum / Difference $\rightarrow$ **Cosine double angle**

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \rightarrow \cos 2\theta = \cos(\theta+\theta) = \cos^2\theta - \sin^2\theta$

Cosine $\rightarrow$ **Double angle** $\rightarrow$ **Power reduction** $\rightarrow$ **Half angle**

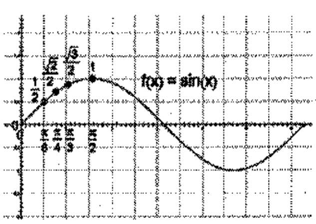
substitute for $\cos^2\theta \rightarrow \cos 2\theta = 1 - 2\sin^2\theta \rightarrow \sin^2\theta = \dfrac{1-\cos 2\theta}{2} \rightarrow \sin\theta = \pm\sqrt{\dfrac{1-\cos 2\theta}{2}}$

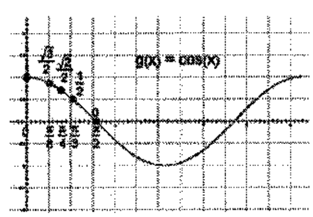
substitute for $\sin^2\theta \rightarrow \cos 2\theta = 2\cos^2\theta - 1 \rightarrow \cos^2\theta = \dfrac{1+\cos 2\theta}{2} \rightarrow$

$\cos\theta = \pm\sqrt{\dfrac{1+\cos 2\theta}{2}}$

Sine: Sum / Difference $\rightarrow$ **Sine double angle**

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha \rightarrow \sin 2\theta = \sin(\theta+\theta) = 2\sin\theta\cos\theta$





"π" is a special # and $\pi \approx 3.142$

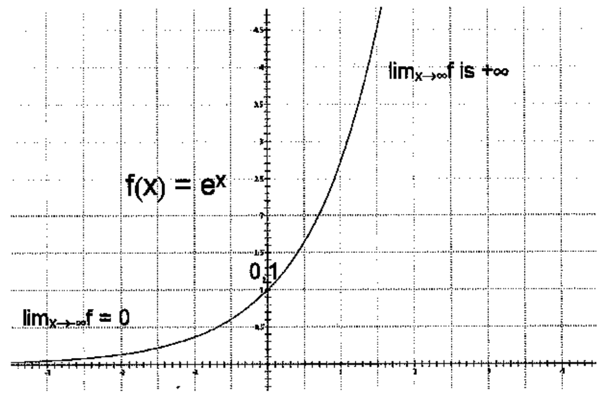
Exponential and Logarithmic Functions

Exponential and logarithmic with the same base are inverse functions (reflect over the $y = x$ line)

Exponential function: $f(x) = a^x$, $Domain: x \in \Re$, $Range: y > 0$

$$\lim_{x \to -\infty} a^x = 0, \quad \lim_{x \to \infty} a^x = \infty$$

Properties of Exponential functions:



$$a^n + a^m = a^n + a^m$$

$$a^n \cdot a^m = a^{n+m}$$

$$\left(a^n\right)^m = a^{nm}$$

$$\left(a^n b^m\right)^k = a^{kn} b^{km}$$

$$a^{-n} = \frac{1}{a^n}$$

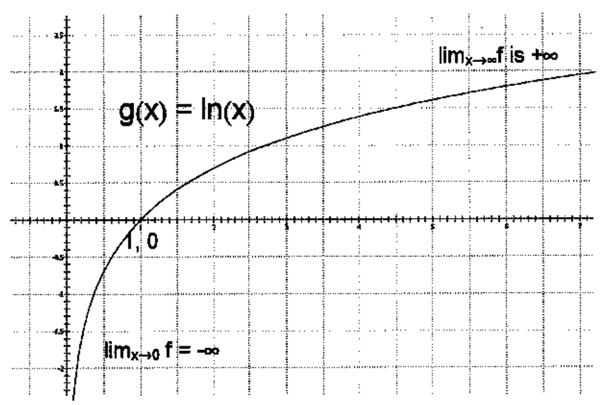
$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$a^0 = 1$$

Logarithmic function: $f(x) = \log_n a$, $Domain: x > 0$, $Range: y \in \Re$

$$\lim_{x \to 0^+} \log_n a = -\infty, \quad \lim_{x \to \infty} \log_n a = \infty$$

Properties of Logarithmic functions:



$$y = \log_a x \Leftrightarrow x = a^y$$

$$\log_a (nm) = \log_a n + \log_a m$$

$$\log a\left(\frac{n}{m}\right) = \log_a n - \log_a m$$

$$\log_a n^m = m \log_a n$$

$$\log_n m = \frac{\log_a m}{\log_a n}$$

$$\log_{10} n \Leftrightarrow \log n$$

$$\log_e n \Leftrightarrow \ln n$$

$$\log_n 1 = 0; \quad \ln e = 1; \quad \log 10 = 1$$

Note: $\log x$ is understood to be $\log_{10} x$ and $\ln x$ is $\log_e x$

"e" is special base and $e \sim 2.718$