Parallel Lines

Hon. Geometry
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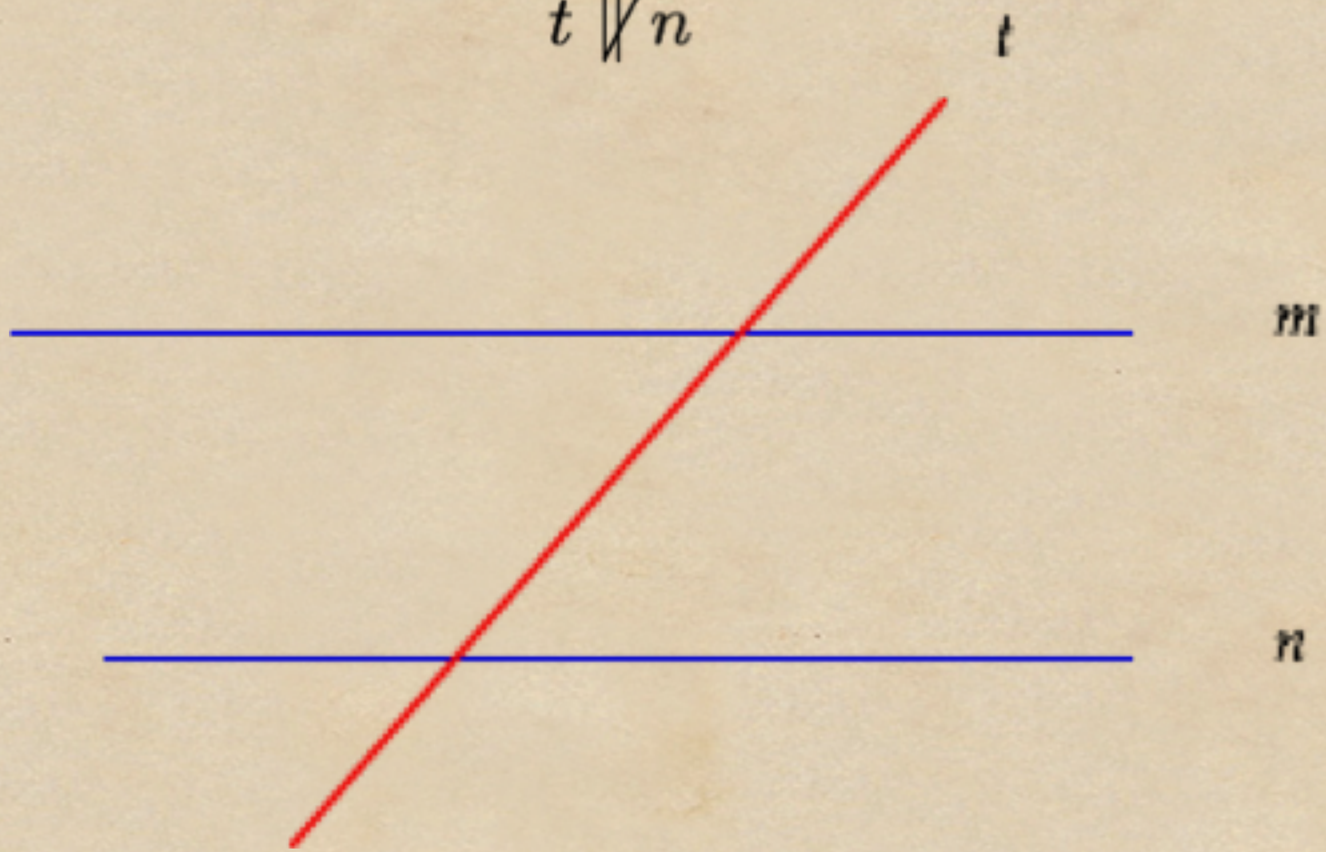
2016

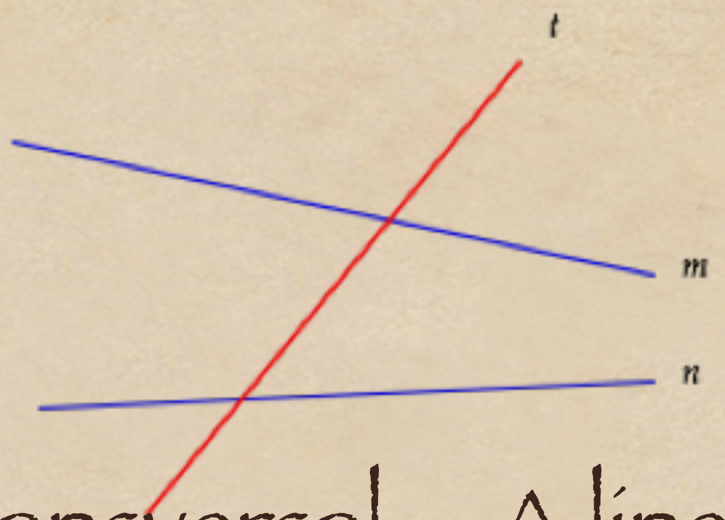
Definitions

- Parallel-- coplanar lines that never intersect
- Skew--lines that never intersect because they are not coplanar

$m \parallel n$

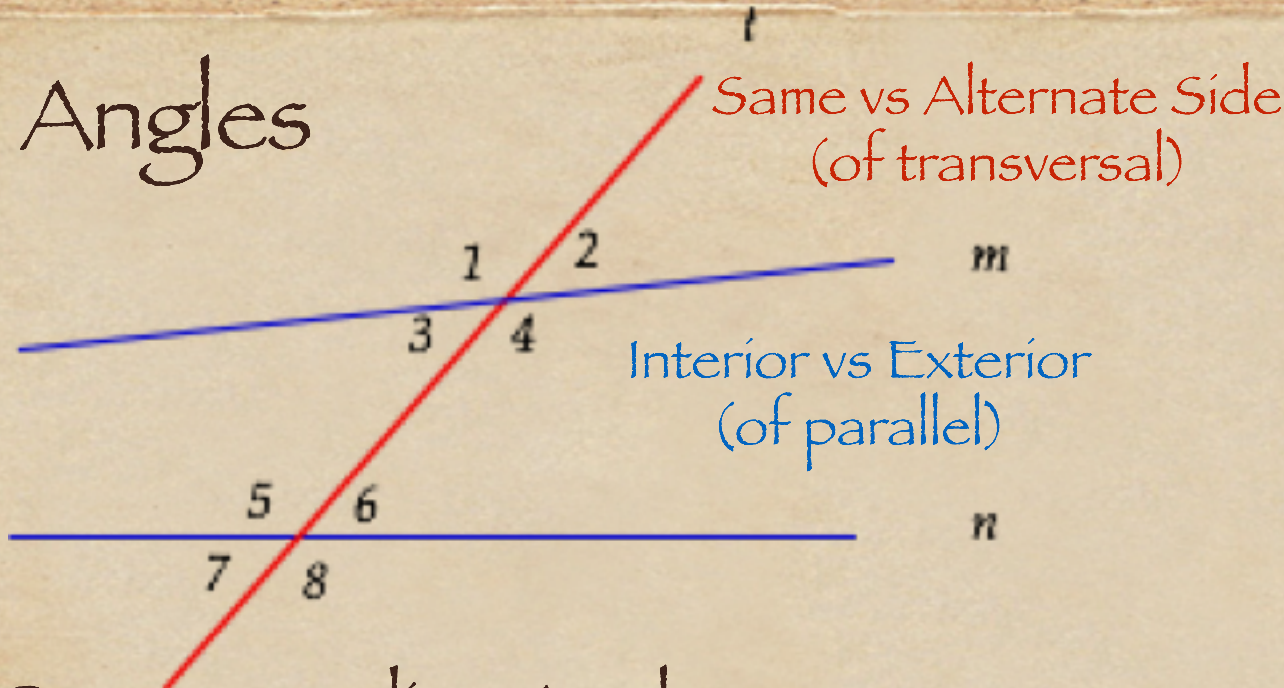
$t \nparallel n$





Transversal-- A line that goes across 2 or more lines

Angles



Corresponding Angles

Iff $m||n, \angle 1 \cong \angle 5 \;, \angle 2 \cong \angle 6 \;, \angle 3 \cong \angle 7 \;, \angle 4 \cong \angle 8$

Consecutive (Same Side) Interior Angles

Iff $m||n, \angle 4 + \angle 6 = 180°$ Iff $m||n, \angle 3 + \angle 5 = 180°$

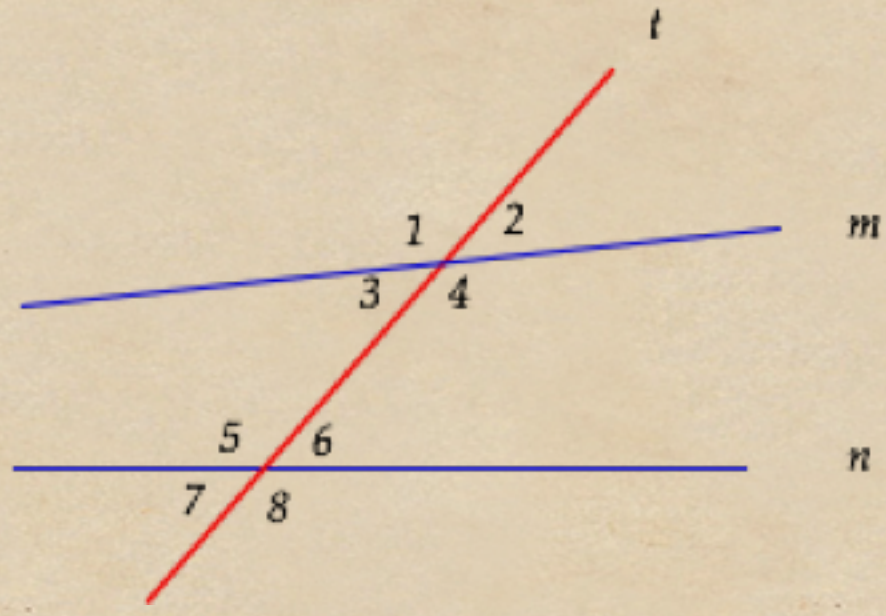
Alternate Interior Angles

Iff $m||n, \angle 3 \cong \angle 6$ Iff $m||n, \angle 4 \cong \angle 5$

Alternate Exterior Angles

Iff $m||n, \angle 1 \cong \angle 8$ Iff $m||n, \angle 2 \cong \angle 7$

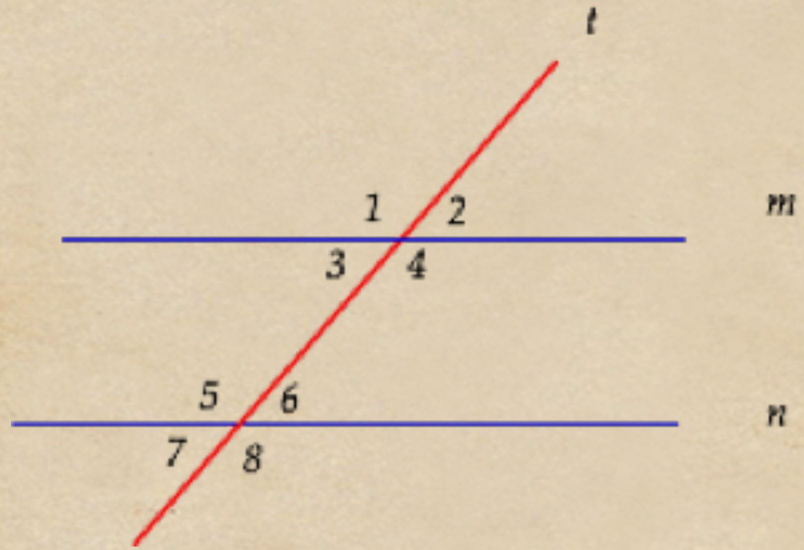
Euclid's 5th Postulate

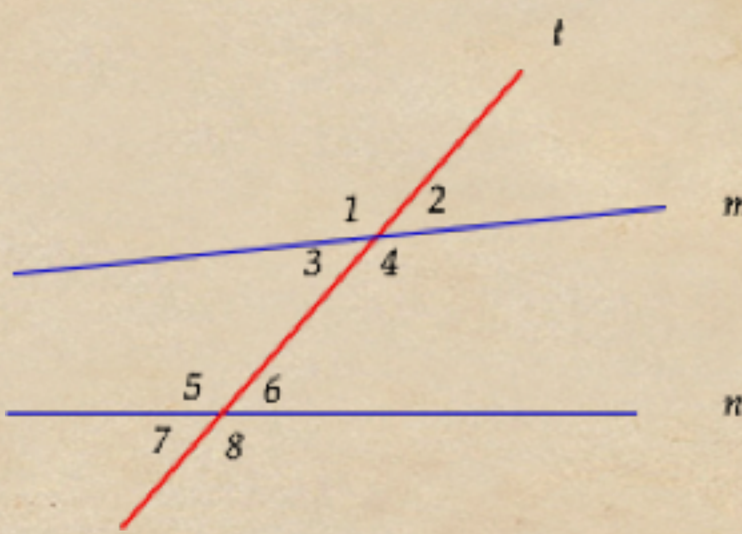


If consecutive (same side) interior angles are not supplementary, the lines are not parallel

Our Book's approach

Postulate:



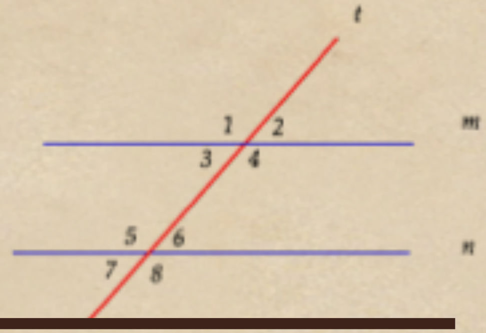


If and only if the lines are parallel, then Corresponding angles are congruent

Theorem: If and only if the lines are parallel, then Alternate Interior Angles are Congruent

Given: $m \parallel n$

Prove: $\angle 3 \cong \angle 6$



$\angle 3 \cong \angle 7$

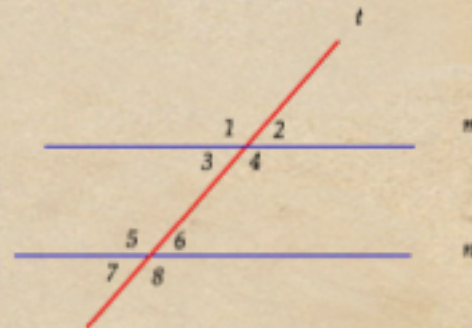
$\angle 6 \cong \angle 7$

$\angle 3 \cong \angle 6$

Alt Int $\angle$'s are $\cong$

Given: $\angle 3 \cong \angle 6$

Prove: $m \parallel n$



$\angle 3 \cong \angle 6$

$\angle 6 \cong \angle 7$

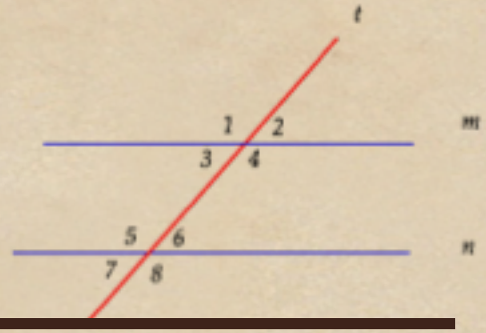
$\angle 3 \cong \angle 7$

$m \parallel n$

Theorem: If and only if the lines are parallel, then Consecutive (Same Side) Interior Angles are Supplementary

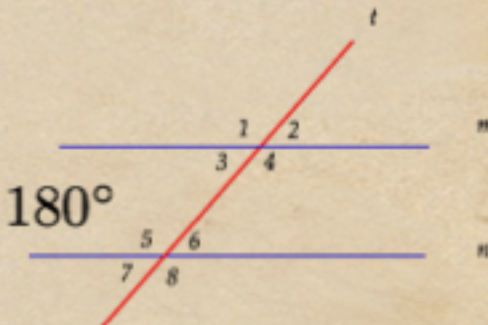
Given: $m \parallel n$

Prove: $\angle 3 + \angle 5 = 180°$



$\angle 3 + \angle 1 = 180°$	
$\angle 5 = \angle 1$	
$\angle 3 + \angle 5 = 180°$	

CI∠'s are Supp



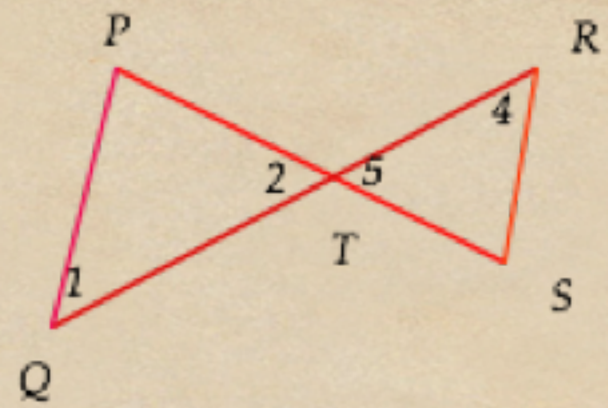
Given: $\angle 3 + \angle 5 = 180°$

Prove: $m \parallel n$

$\angle 3 + \angle 5 = 180°$	
$\angle 3 + \angle 1 = 180°$	
$\angle 3 + \angle 5 = \angle 3 + \angle 1$	
$\angle 5 = \angle 1$	

$m \parallel n$

Example:



Given: $\angle 1 \cong \angle 2; \angle 4 \cong \angle 5$
Prove: $\overline{PQ} \parallel \overline{RS}$

$\angle 1 \cong \angle 2$	Given
$\angle 2 \cong \angle 5$	Vert. $\angle$'s are $\cong$
$\angle 5 \cong \angle 4$	Given
$\angle 1 \cong \angle 4$	Transitive
$\overline{PQ} \parallel \overline{RS}$	Alt Int $\angle$'s s are $\cong$