Mini-Lecture 1.1
Linear Equations

Learning Objectives:

1. Solve a linear equation
2. Solve equations that lead to linear equations
3. Solve problems that can be modeled by linear equations

Examples:

1. $(a)\ 6+3x=9x+6$ $\quad (b)\ 2x-(3x+3)=2x-18$ $\quad (c)\ \dfrac{x+4}{2}+\dfrac{x+1}{3}=10$

2. $(a)\ \dfrac{1}{4}+\dfrac{6}{x}=\dfrac{5}{8}$ $\quad (b)\ x(2x-5)=(2x+2)(x-3)$

   $(c)\ \dfrac{x}{x-6}+1=\dfrac{6}{x-6}$ $\quad (d)\ \dfrac{5}{x-2}=\dfrac{-3}{x+2}+\dfrac{28}{(x-2)(x+2)}$

3. A total of \$51,000 is to be invested, some in bonds and some in certificates of deposit (CDs). If the amount invested in bonds is to exceed that in CDs by \$3,000, how much will be invested in each type of investment?

4. Shannon, who is paid time-and-a-half for hours worked in excess of 40 hours, had gross weekly wages of $\$608$ for 56 hours worked. What is her regular hourly wage?

Teaching Notes:

- Emphasize the need for students to check their answers in the original equation.
- Remind students about the order of operations. For example, $5-2(x+4)\neq 3(x+4)$.
- Review finding the least common multiple.
- Remind students when multiplying through by the LCM to multiply every term on both sides of the equation. When one of the terms is an integer, many students will forget to multiply that term by the LCM.
- Encourage students to find the domain before attempting to solve the rational equation. This may remind them to look for extraneous solutions.
- Refer students to "Steps for Solving Applied Problems" in the textbook.
- When solving formulas for a particular variable, refer students back to "Procedures That Result in Equivalent Equations."

- Many times when solving a formula for a variable inside parentheses, students will divide rather than using the distributive property first. It is helpful to show the solution by both methods so that students can see the differences in the two forms of the answer.

<u>Answers</u>:

1. ***(a)*** $x = 0$ ***(b)*** $x = 5$ ***(c)*** $x = \frac{46}{5}$

2. ***(a)*** $x = 16$ ***(b)*** $x = 6$ ***(c)*** *No solution* ***(d)*** $x = 3$

3. $\$24,000$ in CDs; $\$27,000$ in bonds

4. $\$9.50$/hour

Mini-Lecture 1.2
Quadratic Equations

Learning Objectives:

1. Solve a quadratic equation by factoring
2. Solve a quadratic equation by the square root method.
3. Solve a quadratic equation by completing the square
4. Solve a quadratic equation using the quadratic formula
5. Solve problems that can be modeled by quadratic equations

Examples:

1. Find the real solutions by factoring: $3x^2 + 4x - 4 = 0$.

2. Find the real solutions by using the square root method: $(4x-1)^2 - 16 = 0$.

3. Find the real solutions by completing the square: $x^2 + 4x - 10 = 0$.

4. Find the real solutions by using the quadratic formula: $3x^2 - 5x - 7 = 0$.

5. A ball is thrown vertically upward from the top of a building 48 feet tall with an initial velocity of 32 feet per second. The distance *s* (in feet) of the ball from the ground after *t* seconds is $s = 48 + 32t - 16t^2$.
   (a) After how many seconds does the ball strike the ground?
   (b) After how many seconds will the ball pass the top of the building on its way down?

Teaching Notes:

- Work an example like $x^2 = 25$ by factoring and by the square root method. This will help students remember that there are two solutions when the square root method is used. Many times students will only give one solution when using the square root method.
- Many times completing the square will reveal the difficulties that students have with fractions.
- Show the derivation of the quadratic formula. This reinforces completing the square and clarifies why the quadratic formula works.
- Emphasize the use of the discriminant when solving a quadratic equation using the quadratic formula.
- Warn students to be careful when working with the signs under the radical in the quadratic formula.
- When students use the quadratic formula, they will have trouble simplifying the rational expression. For example, $\frac{10 \pm 5\sqrt{10}}{10} = 1 \pm 5\sqrt{10}$ is a common error.

Answers:

1. $\frac{2}{3}; -2$

2. $\frac{5}{4}; -\frac{3}{4}$

3. $-2 \pm \sqrt{14}$

4. $\frac{5 \pm \sqrt{109}}{6}$

5. ***(a)*** 3 seconds ***(b)*** 2 seconds

Mini-Lecture 1.3
Complex Numbers; Quadratic Equations in the Complex Number System

<u>Learning Objectives</u>:

1. Add, subtract, multiply, and divide complex numbers
2. Solve quadratic equations in the complex number system

<u>Examples</u>:

1. Write each expression in the standard form $a+bi$.
   $(a)\ (2-9i)+(9+7i)$ $(b)(2-4i)-(5+2i)$ $(c)\ (7-4i)(2+i)$
   $(d)\ \dfrac{4}{7-4i}$ $(e)\ \dfrac{6-i}{7+i}$ $(f)\ i^{18}$ $(g)\ (1+i)^3$

2. Perform the indicated operation and express the answer in the form $a+bi$.
   $(a)\ \sqrt{-100}$ $(b)\ \sqrt{(2+5i)(2-5i)}$

3. Solve each equation in the complex number system.
   $(a)\ x^2+5=0$ $(b)\ x^2+2x+7=0$
   $(c)\ 2x^2-4x-5=0$ $(d)\ x^2-2x+5=0$

<u>Teaching Notes</u>:

- Many times students will not remember to express square roots of negative numbers in terms of *i* before multiplying. In the beginning have students simplify as follows: $\sqrt{-25}\sqrt{-4}=\sqrt{-1\bullet 25}\sqrt{-1\bullet 4}=\sqrt{-1}\sqrt{25}\bullet\sqrt{-1}\sqrt{4}=i\sqrt{25}\bullet i\sqrt{4}=$ $i^2\sqrt{100}= -10$
- Many students have problems with signs in complex number problems. Remind them often to be careful with the signs when working with i^2.
- Emphasize that $(a+bi)(a-bi)=a^2+b^2$. Discourage the use of the FOIL method when multiplying a complex number by its conjugate.
- Emphasize the pattern in the powers of i.
- Review the discriminant and the character of the solutions of a quadratic equation.

Answers:

1. ***(a)*** $11-2i$ ***(b)*** $-3-6i$ ***(c)*** $18-i$ ***(d)*** $\frac{28}{65}+\frac{16}{65}i$ ***(e)*** $\frac{41}{50}-\frac{13}{50}i$
   (f) -1 ***(g)*** $-2+2i$

2. ***(a)*** $10i$ ***(b)*** $\sqrt{29}$

3. ***(a)*** $x=\pm\sqrt{5}i$ ***(b)*** $x=-1\pm\sqrt{6}i$ ***(c)*** $x=\frac{2\pm\sqrt{14}}{2}$ ***(d)*** $x=1\pm 2i$

Mini-Lecture 1.4
Radical Equations; Equations Quadratic in Form; Factorable Equations

<u>Learning Objectives</u>:

1. Solve radical equations
2. Solve equations quadratic in form
3. Solve equations by factoring

<u>Examples</u>:

1. Find the real solutions of each equation.
   $(a)\ \sqrt{2x-4}=4$ $(b)\ \sqrt{7-6x}=x$ $(c)\ x=2\sqrt{6x-36}$
   $(d)\ \sqrt{x^2-x-7}=x+3$ $(e)\ \sqrt{3x+1}-\sqrt{x-1}=2$ $(f)\ (3x+3)^{1/2}=9$

2. Find the real solutions of each equation.
   $(a)\ 14x^4-5x^2-1=0$ $(b)\ (x+6)^2+3(x+6)+2=0$ $(c)\ x+\sqrt{x}=30$
   $(d)\ \dfrac{1}{(x+6)^2}=\dfrac{1}{(x+6)}+12$ $(e)\ 8x^{2/3}-39x^{1/3}-5=0$

3. Find the real solutions of each equation by factoring.
   $(a)\ x^3-49x=0$ $(b)\ 7x^3=2x^2$ $(c)\ x^3-14x^2+48x=0$ $(d)\ x^3+x^2-25x-25=0$

<u>Teaching Notes</u>:

- Radical equations such as part d in problem 1 above will result in such mistakes as $\sqrt{x^2-x-7}=x+3 \Rightarrow x^2-x-7=x^2+9$. Students will square both sides incorrectly. Make sure you go over this.
- Suggest that students review factoring by grouping.
- Emphasize the necessity of checking answers in radical equations.
- If you use substitution to solve problems such as those in problem 2 above, make sure you reiterate the necessity of substituting back and continuing on to find the true solution. Again, make them check their answers.

<u>Answers</u>:

1. ***(a)*** 10 ***(b)*** 1 ***(c)*** 12 ***(d)*** $-\frac{16}{7}$ ***(e)*** $1, 5$ ***(f)*** 26

2. ***(a)*** $-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$ ***(b)*** $-7, -8$ ***(c)*** 25 ***(d)*** $-\frac{19}{3}, -\frac{23}{4}$ ***(e)*** $-\frac{1}{512}, 125$

3. ***(a)*** $-7, 0 - 7$ ***(b)*** $0, \frac{2}{7}$ ***(c)*** $0, 6, 8$ ***(d)*** $-1, -5, 5$

Mini-Lecture 1.5
Solving Inequalities

Learning Objectives:

1. Use interval notation
2. Use properties of inequalities
3. Solve inequalities
4. Solve combined inequalities

Examples:

1. Write each inequality using interval notation.
   $(a)\ -4 < x \le 5$ $\quad (b)\ 0 < x < 4$ $\quad (c)\ 5 \le x < 12$ $\quad (d)\ 8 \le x \le 16$

2. Write each interval as an inequality involving x.
   $(a)\ (-5,10)$ $\quad (b)\ [4,16]$ $\quad (c)\ [0,6)$ $\quad (d)\ (-5,1]$

3. Solve each inequality. Express the answer in interval notation.
   $(a)\ x+7<1$ $\quad (b)\ 3x-9 \ge 3+x$ $\quad (c)\ 8-7(1-x) \le 7$ $\quad (d)\ \frac{x}{4} \ge 3-\frac{x}{16}$

4. Solve each inequality. Express the answer in interval notation.
   $(a)\ -15 \le 9-4x \le 25$ $\quad (b)\ -4 < \frac{4x-8}{5} < 0$ $\quad (c)\ -2 < 1-\frac{1}{2}x < 5$
   $(d)\ (x+5)(x-7) > (x-5)(x+5)$ $\quad (e)\ (8x+4)^{-1} < 0$ $\quad (f)\ 0 < \frac{5}{x} < \frac{8}{9}$

Teaching Notes:

- The main problem that students will have is not reversing the inequality sign when multiplying or dividing by a negative number.
- Discourage the use of open and closed circles on graphs. Using parentheses and brackets reinforces the concepts of interval notation.
- Emphasize Table 1 in the book, which summarizes interval notation, inequality notation, and their graphs.
- Stress the Multiplication Properties for Inequalities.
- Many students will solve inequalities incorrectly if the variable is on the right side (Ex. $-2 \ge x$). Encourage them to flip the inequality around before writing the interval notation. See "Procedures That Leave the Inequality Symbol Unchanged" and "Procedures That Reverse the Sense or Direction of the Inequality Symbol."

Answers:

1. ***(a)*** $(-4,5]$ ***(b)*** $(0,4)$ ***(c)*** $[5,12)$ ***(d)*** $[8,16]$

2. ***(a)*** $-5 < x < 10$ ***(b)*** $4 \le x \le 16$ ***(c)*** $0 \le x < 6$ ***(d)*** $-5 < x \le 1$

3. ***(a)*** $(-\infty,-6)$ ***(b)*** $[6,\infty)$ ***(c)*** $\left(-\infty,\frac{6}{7}\right]$ ***(d)*** $\left[\frac{48}{5},\infty\right)$

4. ***(a)*** $[-4,6]$ ***(b)*** $(-3,2)$ ***(c)*** $(-8,6)$ ***(d)*** $(-\infty,-5)$ ***(e)*** $\left(-\infty,-\frac{1}{2}\right)$ ***(f)*** $\left(\frac{45}{8},\infty\right)$

Mini-Lecture 1.6
Equations and Inequalities Involving Absolute Value

Learning Objectives:

1. Solve equations involving absolute value
2. Solve inequalities involving absolute value

Examples:

1. Solve each equation.

   $(a)\ |5x-10|=15$ $(b)\ \left|\frac{2}{3}x+6\right|=12$ $(c)\ |4-3x|-4=1$ $(d)\ |x^2+x-1|=1$

2. Solve each absolute value inequality.

   $(a)\ |3x|\le 21$ $(b)\ |4x-3|\ge 9$ $(c)\ |2-6x|-5<1$ $(d)\ -|3x-3|\ge -8$

Teaching Notes:

- When solving absolute value equations, students will sometimes forget that there are two solutions.
- Students often will not isolate the absolute value expression before trying to solve, such as examples 1c and 2c above.
- Do not be surprised if you see an answer such as $-3<x>2$. Students will invariably try to combine two intervals that cannot be combined.
- Do not allow students to break an absolute value inequality in the form $|X|\le c$ (or $|X|<c$) into two separate inequalities (Ex. $|x+1|<2$ written as $x+1>-2$ and $x+1<2$). This does not reinforce the concept of intersection.

Answers:

1. ***(a)*** $x\in\{-1,5\}$ ***(b)*** $x\in\{-27,9\}$ ***(c)*** $x\in\left\{-\frac{1}{3},3\right\}$ ***(d)*** $x\in\{-2,-1,0,1\}$

2. ***(a)*** $[-7,7]$ ***(b)*** $\left(-\infty,-\frac{3}{2}\right]\cup[3,\infty)$ ***(c)*** $\left(-\frac{2}{3},\frac{4}{3}\right)$ ***(d)*** $\left[-\frac{5}{3},\frac{11}{3}\right]$

Mini-Lecture 1.7
Problem Solving: Interest, Mixture, Uniform Motion, Constant Rate Job Applications

Learning Objectives:

1. Translate verbal descriptions into mathematical expressions
2. Solve interest problems
3. Solve mixture problems
4. Solve uniform motion problems
5. Solve constant rate job problems

Examples:

1. Translate the following sentence into a mathematical equation.
   "The area, A, of a circle is the product of the number π and the square of the radius, r.

2. Betsy, a recent retiree, requires $\$6,000$ per year in extra income. She has $\$70,000$ to invest and can invest in B-rated bonds paying 17% per year or in a CD paying 7% per year. How much money should be invested in each to realize exactly $\$6,000$ in interest per year?

3. A nut store normally sells cashews for $\$4$ per pound and peanuts for $\$1.50$ per pound. At the end of the month the peanuts had not sold well, so, in order to sell 60 pounds of peanuts, the manager decided to mix the 60 pounds of peanuts with some cashews and sell the mixture for $\$2.50$ per pound. How many pounds of cashews should be mixed with the peanuts to ensure no change in the profit?

4. A boat can maintain a constant speed of 34 mph relative to the water. The boat makes a trip upstream to a certain point in 21 minutes; the return trip takes 13 minutes. What is the speed of the current?

5. Trent can deliver his newspapers in 60 minutes. It takes Lois 40 minutes to do the same route. How long would it take them to deliver the newspapers if they work together?

Teaching Notes:

- It is important that you show students the patterns for each type of problem.
- Emphasize the use of Figure 15 which illustrates the modeling process.
- Encourage the use of drawings and charts when solving mixture and uniform motion problems.
- Many students understand that $8 - x \neq x - 8$, but will confuse "a number decreased by 8" with "8 decreased by a number".

- Emphasize that solving an equation and answering the problem's question may not be the same thing.
- Some students will set up equations with quantity on one side of an equation and monetary value on the other side of the equation. Emphasize that the units on both sides of the equation must be the same.

Answers:

1. $A = \pi r^2$

2. \$11,000 at 17% and \$59,000 at 7%

3. 40 pounds of cashews

4. 8 mph

5. 24 minutes