Mini-Lecture 2.1
The Distance and Midpoint Formulas

Learning Objectives:

1. Use the Distance Formula
2. Use the Midpoint Formula

Examples:

1. Find the distance between the points $(-3,7)$ and $(4,10)$.

2. Determine whether the triangle formed by points $A=(-2,2)$, $B=(-2,1)$, and $C=(5,4)$ form a right triangle.

3. Find the midpoint of the line segment joining the points $P_1=(6,-3)$ and $P_2=(4,2)$.

Teaching Notes:

- Go over the terms used in introducing the rectangular coordinate system.
- Tell students the distance formula will be used in several applications later in the course.
- Remind students to be careful with signs when using the distance formula.
- Some students will use subtraction in the midpoint formula, rather than addition. Emphasize that the coordinates of the midpoint are the averages of the x-coordinates and the y-coordinates.

Answers:

1. $\sqrt{58}$ 2. No: $|AB|^2=5,\ |BC|^2=34,\ |AC|^2=53$ 3. $\left(5,-\frac{1}{2}\right)$

Mini-Lecture 2.2
Graphs of Equations in Two Variables; Intercepts; Symmetry

Learning Objectives:

1. Graph equations by plotting points
2. Find intercepts from a graph
3. Find intercepts from an equation
4. Test for an equation for symmetry with respect to the x-axis, the y-axis, and the origin
5. Know how to graph key equations

Examples:

1. Determine whether the points $(0,3)$, $(-2,0)$, and $(2,7)$ are on the graph of the equation $y = x^3 - 2x + 3$.

2. Find the intercepts of the equation $y = 2x - 1$ by plotting points.

3. List the intercepts, test for symmetry, and graph each equation.

   $(a)\ y^2 - x - 4 = 0$ $\quad (b)\ y = \dfrac{x}{x^2 - 4}$

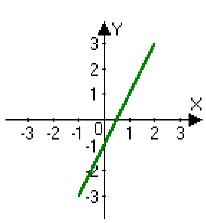
Teaching Notes:

- When graphing by plotting points, be sure to emphasize that the *y*-coordinate is determined by the value of *x*. This will help establish the function concept later.
- Emphasize the graphing of the key functions. It is important that students know the basic shapes of these graphs when this topic is revisited later in the course.
- Encourage students to find the intercepts and to test for symmetry before beginning to graph.
- Emphasize "Procedure for Finding Intercepts" in the book.
- Emphasize "Tests for Symmetry" in the book.
- When testing for symmetry, remind students that a negative number raised to an even power is positive and a negative number raised to an odd power is negative.

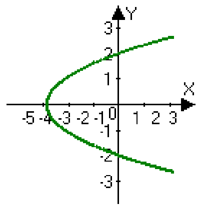
Answers:

1. Yes, No, Yes

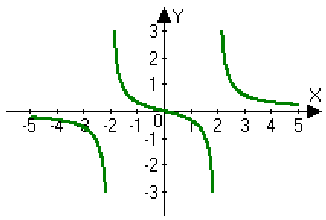
2. $\left(\frac{1}{2},0\right),(0,-1)$



3. ***(a)*** $(0,2),(0,-2),(-4,0)$ Symmetric with respect to the x-axis



(b) $(0,0)$ Symmetric with respect to the origin



Mini-Lecture 2.3
Lines

Learning Objectives:

1. Calculate and interpret the slope of a line
2. Graph lines given a point and the slope
3. Find the equation of a vertical line
4. Use the point-slope form of a line; identify horizontal lines
5. Use the slope-intercept form of a line
6. Find the equation of a line given two points
7. Graph lines written in general form using intercepts
8. Find equations of parallel lines
9. Find equations of perpendicular lines

Examples:

1. Determine the slope of the line containing the points $(-5,4)$ and $(0,7)$.

2. Graph the line containing the point $(2,4)$ with slope $m=\frac{-2}{3}$.

3. Write an equation of the line satisfying the given conditions:
   (a) Slope = $\frac{3}{4}$, containing the point $(-2,4)$ *(b)* Containing the points $(4,2)$ and $(3,-4)$.
   (c) x-intercept = 3, y-intercept = -2 *(d)* Vertical line containing $(5,-1)$.
   (e) Parallel to the line $3x-4y=5$ and containing the point $(3,-6)$.

4. Find the slope and y-intercept of the line $4x-6y=-3$.

5. Find the intercepts and graph the line $-2x+y=4$.

Teaching Notes:

- Emphasizing that the definition of the slope is "the change in y over the change in x" will help students remember the formula for the slope.
- Showing the students how to derive the point-slope formula from the slope formula will help students remember the point-slope form.
- Emphasize care with signs when using both the slope formula and the point-slope formula
- When using the slope and y-intercept to graph a line, encourage students to count down and to the right when the slope is negative. Consistency will help students successfully graph all lines.
- Emphasize the slopes and equations of horizontal and vertical lines.

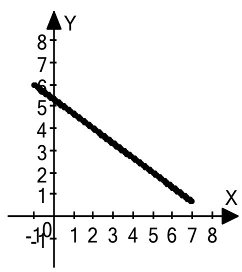
- Use the graphing utility and graph several lines on the same square screen. This will help students see the role that the slope plays in graphing lines.
- Emphasize the relationship between the slopes of parallel lines.

- Emphasize the relationship between the slopes of perpendicular lines, especially emphasize that the slopes are negative reciprocals of each other. Many students will remember the fact that the slopes are reciprocals, but will forget that they have opposite signs.

<u>**Answers**</u>:

1. $m = \dfrac{7-4}{0-(-5)} = \dfrac{3}{5}$

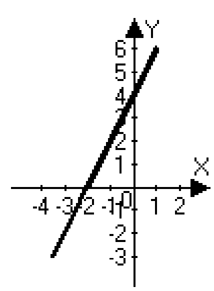
2.



3. ***(a)*** $y = \frac{3}{4}x + \frac{11}{2}$ ***(b)*** $y = 6x - 22$ ***(c)*** $y = \frac{2}{3}x - 2$ ***(d)*** $x = 5$ ***(e)*** $y = \frac{3}{4}x - \frac{33}{4}$

4. Slope $= \dfrac{2}{3}$; y-intercept $= \dfrac{1}{2}$

5. x-intercept = -2, y-intercept = 4



Mini-Lecture 2.4
Circles

Learning Objectives:

1. Write the standard form of the equation of a circle
2. Graph a circle
3. Work with the general form of the equation of a circle

Examples:

1. Write the standard form and general form of the equation of each circle with radius r and center (h,k). Graph each circle.

   $(a)\ r=3;\ (h,k)=(-2,3).$ $\quad (b)\ r=\frac{2}{3};\ (h,k)=(0,0).$

2. Find the center (h,k) and radius r of each circle.

   $(a)\ 2(x-2)^2+2(y+3)^2=8$ $\quad (b)\ x^2+y^2-6x+2y+4=0$

3. Find the general form of the equation of each circle.

   (a) Center $(2,-3)$ and containing the point $(0,4)$.

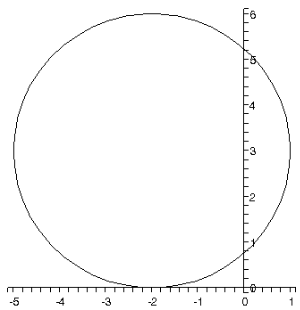
   (b) Endpoints of a diameter at $(6,10)$ and $(-4,-4)$.

Teaching Notes:

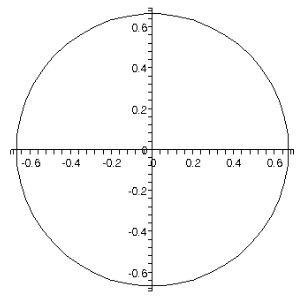
- Emphasize taking the opposite signs of those in the parentheses when finding the center of a circle.
- Some students will not recognize that a circle of the form $x^2+y^2=r^2$ has the origin as its center. Show them this form written as $(x-0)^2+(y-0)^2=r^2$.
- Many students will forget to add the same numbers to both sides of the equation when completing the squares in the equation of a circle.
- Emphasize the difference between the standard form of the equation of a circle and the general form.
- Have students review the method of completing the square.

Answers:

1. ***(a)*** $(x+2)^2+(y-3)^2=9;\ x^2+y^2+4x-6y+4=0$ ***(b)*** $x^2+y^2=\frac{4}{9};\ x^2+y^2-\frac{4}{9}=0$



1. ***(a)***



1. ***(b)***

2. ***(a)*** $c=(2,-3);\ r=2$ ***(b)*** $c=(3,-1);\ r=\sqrt{6}$

3. ***(a)*** $(x-2)^2+(y+3)^2=53$ ***(b)*** $(x-1)^2+(y-3)^2=74$

Mini-Lecture 2.5
Variation

Learning Objectives:

1. Construct a model using direct variation
2. Construct a model using inverse variation
3. Construct a model using joint variation

Examples:

1. The monthly payment p on a mortgage varies directly with the amount borrowed B. If the monthly payment on a 30-year mortgage is $\$5.75$ for every $\$1000$ borrowed, find a function $p=p(B)$ that relates the monthly payment p to the amount borrowed B for a mortgage with the same terms. Then find the monthly payment p when the amount borrowed B is $\$225,000$.

2. The length of a violin string varies inversely as the frequency of its vibrations. If a string 8 inches long vibrates at a frequency of 640 cycles per second, what is the frequency of a string that is 10 inches long?

3. The volume of a cone V varies jointly as its height h and the square of its radius r. A cone with a radius of 4 cm, and a height of 6 cm, has a volume of 32π. Find the volume of a cone having a radius of 15 cm and a height of 30 cm.

Teaching Notes:

- Remind students to always find the constant of variation first, if enough information is given in the problem.
- Emphasize the graphs in the figures that accompany the examples on direct and inverse variation in the book. These graphs will help the student visualize the type of variation.
- Tell students to read each problem carefully. A common mistake is to read *square root* as *squared*.

Answers:

1. $\$1293.75$

2. 512 cycles per second

3. $2250\pi\ \text{cm}^3$