Mini-Lecture R.1
Real Numbers

Learning Objectives:

1. Work with sets
2. Classify numbers
3. Evaluate numerical expressions
4. Work with properties of real numbers

Examples:

1. Use $A=\{0,1,6,7,9\}, B=\{2,3,8,9\},$ and $C=\{0,1,2,4,6,7\}$ to find the set $(A\cup B)\cap C.$
2. Use $U=$ universal set $=\{0,1,2,3,4,5,6,7,8,9\}$, $B=\{4,6,7,8,9\}$, and $C=\{1,4,5,6,9\}$ to find the set $\overline{B\cap C}$.
3. List the numbers in the set $\left\{8,\frac{3}{4},\sqrt{5},\pi,1,-9\right\}$ that are *(a)* Natural numbers, *(b)* Integers, *(c)* Rational numbers, *(d)* Irrational numbers, *(e)* Real numbers.
4. Approximate 86.7339: *(a)* rounded and *(b)* truncated to three decimal places.
5. Evaluate: *(a)* $9-\left[4\cdot 5+2\cdot(10-9)\right]$ *(b)* $\dfrac{\left(\frac{17}{20}\right)}{\left(\frac{11}{12}\right)}$
6. Use the Distributive Property to remove the parentheses from $(x-3)(x+1)$.

Teaching Notes:

- Many students don't understand what is meant by the term "real number." Going over the number system is worth the time.
- Many students don't understand the difference between rounding or truncating numbers.
- Emphasize the use of Venn diagrams when working with intersection, union, and the complement of sets.

Answers:

1. $\{0,1,2,6,7\}$ 2. $\{0,1,2,3,5,7,8\}$
3. ***(a)*** $1,8$ ***(b)*** $-9,1,8$ ***(c)*** $\frac{3}{4},-9,1,8$ ***(d)*** $\pi,\sqrt{5}$ ***(e)*** $8,\frac{3}{4},\sqrt{5},\pi,1,-9$
4. ***(a)*** 86.734 ***(b)*** 86.733 5. ***(a)*** -13 ***(b)*** $\frac{51}{55}$ 6. x^2-2x-3

Mini-Lecture R.2
Algebra Essentials

Learning Objectives:

1. Graph inequalities
2. Find distance on the real number line
3. Evaluate algebraic expressions
4. Determine the domain of a variable
5. Use the laws of exponents
6. Evaluate square roots
7. Use a calculator to evaluate exponents
8. Use scientific notation

Examples:

1. Replace the question mark by <, >, or =. *(a)* $\frac{8}{9}?0.89$ *(b)* $\frac{5}{6}?0.83$
2. Graph $x \geq -3$ on the number line.
3. On the real number line, what is the distance between -5 and 2?
4. Evaluate if $x = -6$ and $y = 2$: *(a)* $\frac{4x+3y}{6+6y}$ *(b)* $|2x-4y|$
5. Determine the value(s) of x that must be excluded from the domain of the variable in $\frac{x^2+8x-3}{x^3-4x}$.
6. Simplify each expression:

   (a) 10^{-2} *(b)* $8^{-3} \cdot 8$ *(c)* $\sqrt{(-4)^2}$ *(d)* $\left(x^9 y^{-4}\right)^6$ *(e)* $\left(\frac{4x^{-5}}{7x^{-8}}\right)^{-3}$
7. Write in scientific notation: *(a)* 731.3 *(b)* 0.000442.

Teaching Notes:

- When simplifying $\frac{1}{4x^{-1}}$, many students will simplify using $4x$ as the base of -1. Showing the exponent of 1 on the 4, $\left(\frac{1}{4^1 x^{-1}}\right)$, helps clarify this problem.
- Emphasize that $2^2 \bullet 2^3 \neq 4^5$ by expanding both expressions and comparing the results.
- When using scientific notation, remind students that a large exponent on 10 means *large number* and a small exponent on 10 means *small number*.
- Given an expression like $\left(2a^5 b^3\right)^4$, sometimes a student will multiply the numeric coefficient by the exponent, rather than raising it to a power.

Answers:

1. ***(a)*** $<$ ***(b)*** $>$

2. 

3. 7

4. ***(a)*** -1 ***(b)*** 20

5. $x = 0,\ x = -2,\ x = 2$

6. ***(a)*** $\frac{1}{100}$ ***(b)*** $\frac{1}{64}$ ***(c)*** 4 ***(d)*** $\frac{x^{54}}{y^{24}}$ ***(e)*** $\frac{343x^{15}}{64y^{24}}$

7. ***(a)*** 7.313×10^{2} ***(b)*** 4.42×10^{-4}

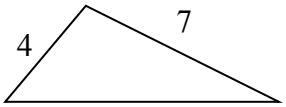
Mini-Lecture R.3
Geometry Essentials

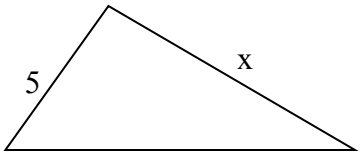
Learning Objectives:

1. Use the Pythagorean Theorem and its converse
2. Know geometry formulas
3. Understand congruent triangles and similar triangles

Examples:

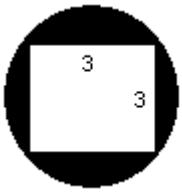
1. The legs of a right triangle have lengths of 9 and 12. Find the hypotenuse.
2. Find the area of a triangle with height 7 inches and a base 8 inches.
3. Find the exact area and circumference of a circle with a radius of 4 meters.
4. Find the volume and surface area of a sphere of radius 9 centimeters.
5. Given that the following are similar triangles, find x.





6. How many feet does a wheel with a diameter of 20 inches travel after 3 revolutions?

7. Find the area of the shaded region if the side of the square is 3 inches.



Teaching Notes:

- Since the Pythagorean Theorem is used extensively throughout mathematics, make sure that students understand it and have them memorize it. Emphasize that it does not apply to triangles that are not right triangles.
- Define "converse" before explaining the converse of the Pythagorean Theorem.
- Students need to know the formulas involving triangles and circles.
- Emphasize "Determining Congruent Triangles" and "Determining Similar Triangles" in the book.

Answers:

1. 15 2. 28 square inches 3. $A = 16\pi$ m^2 $C = 8\pi$ m

4. $V = 972\pi$ cm^3 $SA = 324\pi$ cm^2 5. $x = \frac{35}{4}$ 6. ≈ 15.7 ft 7. $\frac{9}{2}\pi - 9 \approx 5.14$ in^2

Mini-Lecture R.4
Polynomials

Learning Objectives:

1. Recognize monomials
2. Recognize polynomials
3. Add and subtract polynomials
4. Multiply polynomials
5. Know formulas for special products
6. Divide polynomials using long division
7. Work with polynomials in two variables

Examples:

1. State whether the expression is a polynomial. If it is, state the degree.
   $(a)\ 3x^3 - 7x + 2 \quad (b)\ 3x - \dfrac{2}{x} \quad (c)\ \sqrt{2}x^2 - 3\pi \quad (d)\ \sqrt{2x} + 5x$
2. Simplify each expression.
   $(a)\left(7x^3 - 6x^2 + 3x + 8\right) - \left(6x^2 - 2x + 7\right) \quad (b)\ \left(9x^5 + 2x^3 + 8x\right) + \left(5x^4 - 9x^3 + 9x^2\right)$
3. Simplify each expression.
   $(a)\ (x+2)\left(x^2 + 3x - 2\right) \quad (b)\ (5x+3)(x-4) \quad (c)\ (x-2y)(x+y)$
   $(d)\ (x+8)^2 \quad (e)\ (x+2)(x-2) \quad (f)\ (x-2)^3$
4. Divide $5x^4 - 3x^2 + 8x + 6$ by $x^2 + 8$.

Teaching Notes:

- It is important that students understand what a polynomial is.
- Encourage students to learn the Special Products. This knowledge will save them time in future coursework.
- Emphasize that $(A \pm B)^2 \neq A^2 \pm B^2$, $(A \pm B)^3 \neq A^3 \pm B^3$, etc.
- Show that if *n* is the exponent on the binomial, then the product will have $n+1$ terms.
- Show an example of addition, subtraction, and multiplication of polynomials using the vertical method. Many students relate better to this method.
- Be sure to work the example of dividing two integers in the book before dividing two polynomials.

Answers:

1. ***(a)*** Yes; degree 3 ***(b)*** No ***(c)*** Yes; degree 2 ***(d)*** No

2. ***(a)*** $7x^3 - 12x^2 + 5x + 1$ ***(b)*** $9x^5 + 5x^4 - 7x^3 + 9x^2 + 8x$

3. ***(a)*** $x^3 + 5x^2 + 4x - 4$ ***(b)*** $5x^2 - 17x - 12$ ***(c)*** $x^2 - xy - 2y^2$ ***(d)*** $x^2 + 16x + 64$

(e) $x^2 - 4$ ***(f)*** $x^3 - 6x^2 + 12x - 8$ 4. $5x^2 - 43 + \dfrac{8x + 350}{x^2 + 8}$

Mini-Lecture R.5
Factoring Polynomials

Learning Objectives:

1. Factor the difference of two squares, and the sum and difference of two cubes
2. Factor perfect squares
3. Factor a second-degree polynomial : $x^2 + Bx + C$
4. Factor by grouping
5. Factor a second-degree polynomial: $Ax^2 + Bx + C,\ A \neq 1$
6. Complete the square

Examples:

1. Factor: $(a)\ x^2 - 36 \qquad (b)\ x^2 - 49 \qquad (c)\ x^3 - 64$

2. Factor: $(a)\ x^2 + 12x + 36 \qquad (b)\ x^2 - 16x + 64 \qquad (c)\ 81x^2 + 18x + 1$

3. Factor: $(a)\ x^2 - 10x + 24 \qquad (b)\ x^2 - 3x - 54 \qquad (c)\ x^2 + x - 30$

4. Factor: $(a)\ 18x^2 + 12x + 15x + 10 \qquad (b)\ 6x^2 + 21x + 8x + 28$

5. Factor: $(a)\ 36x^2 + 36x + 9 \qquad (b)\ 54x^4y - 16xy^4$
6. Determine the number that should be added to complete the square. Then factor the expression: $r^2 + \frac{1}{4}r$.

Teaching Notes:

- Emphasize that factoring is the reverse of multiplication by checking the factored form.
- Many students will forget to look for the GCF first. Example: $25x^2 - 100y^2$
- Emphasize Steps for Factoring $Ax^2 + Bx + C$ given in this section of the book.
- Show an example of trying to factor a trinomial that is prime.
- Remind students that the order of the terms may need to be changed when factoring by grouping.
- Emphasize that the GCF will have the smallest power. Keeping this in mind will help students later when they factor expressions with fractional and negative exponents.
- Explain Figure 27. It will give students a better understanding of the completing the square process.

Answers:

1. ***(a)*** $(x-6)(x+6)$ ***(b)*** $(x-7)(x+7)$ ***(c)*** $(x-4)(x^2+4x+16)$
2. ***(a)*** $(x+6)^2$ ***(b)*** $(x-8)^2$ ***(c)*** $(9x+1)^2$
3. ***(a)*** $(x-6)(x-4)$ ***(b)*** $(x-9)(x+6)$ ***(c)*** $(x-5)(x+6)$
4. ***(a)*** $(3x+2)(6x+5)$ ***(b)*** $(2x+7)(3x+4)$
5. ***(a)*** $9(2x+1)^2$ ***(b)*** $2xy(3x-2y)(9x^2+6xy+4y^2)$
6. $\frac{1}{64}; \left(x+\frac{1}{8}\right)^2$

Mini-Lecture R.6
Synthetic Division

Learning Objectives:

1. Divide polynomials using synthetic division

Examples:

1. For the given expression, use synthetic division to find the quotient and the remainder.
   (a) $x^5 - x^4 + x^3 - 2x^2 + 3x - 6$ divided by $x + 1$
   (b) $12x^3 + x^2 - \frac{9}{2}x - 1$ divided by $x + \frac{1}{2}$
   (c) $x^4 - 16$ divided by $x + 2$

2. Use synthetic division to determine whether $x - c$ is a factor of the given polynomial.
   $(a)\ 12x^3 - 15x^2 - 27x + 60;\ x - 2$
   $(b)\ 2x^4 - x^3 - 4x + 2;\ x - \frac{1}{2}$
   $(c)\ 9x^6 + 84x^3 + 96;\ x + 2$

Teaching Notes:

- Students need to understand that synthetic division has significant importance in solving equations of degree greater than 2.
- Remind students repeatedly that when dividing using synthetic division, they must put a zero in place of the missing power.
- Remind students repeatedly about the appropriate sign of *c* when using synthetic division to divide by $x - c$.
- Remind students that they can check their answers for synthetic division problems by multiplying the quotient and the divisor and then adding any remainder.

Answers:

1. ***(a)*** $x^4 - 2x^3 + 3x^2 - 5x + 8 - \frac{14}{x+1}$ ***(b)*** $12x^2 - 5x - 2$ ***(c)*** $x^3 - 2x^2 + 4x - 8$

2. ***(a)*** No ***(b)*** Yes ***(c)*** Yes

Mini-Lecture R.7
Rational Expressions

Learning Objectives:

1. Reduce a rational expression to lowest terms
2. Multiply and divide rational expressions
3. Add and subtract rational expressions
4. Use the Least Common Multiple Method
5. Simplify complex rational expressions

Examples:

1. Reduce each rational expression to lowest terms.

$(a)\ \dfrac{y^2-64}{6y^2-36y-96}$ $\quad$ $(b)\ \dfrac{x^2+3x-54}{6-x}$

2. Perform the indicated operation and simplify the result.

$(a)\ \dfrac{3x+21}{9x^7}\cdot\dfrac{x}{x^2-49}$ $\quad$ $(b)\ \dfrac{x^2-3x-10}{x^2+3x-40}\cdot\dfrac{x^2+5x-24}{x^2+10x+16}$ $\quad$ $(c)\ \dfrac{\left(\dfrac{2x}{x^2-49}\right)}{\left(\dfrac{3x-15}{9x+63}\right)}$

$(d)\ \dfrac{\left(\dfrac{x^2+11x+18}{x^2-11x+18}\right)}{\left(\dfrac{x^2+7x-18}{x^2-7x-18}\right)}$ $\quad$ $(e)\ \dfrac{x^2}{5x-7}-\dfrac{4}{5x-7}$ $\quad$ $(f)\ \dfrac{5}{x-8}+\dfrac{x}{8-x}$

$(g)\ \dfrac{x}{x+9}+\dfrac{4x-5}{x-9}$ $\quad$ $(h)\ \dfrac{x}{x^2-8x+7}-\dfrac{x}{x^2-2x-35}$ $\quad$ $(i)\ \dfrac{4x}{x^2-9}-\dfrac{6}{x^2+x-12}$

3. Perform the indicated operation and simplify the result.

$(a)\ \dfrac{3+\dfrac{1}{x}}{6-\dfrac{1}{x}}$ $\quad$ $(b)\ \dfrac{\dfrac{x-6}{x+6}+\dfrac{x-1}{x+1}}{\dfrac{x}{x+1}-\dfrac{7x-3}{x}}$

Teaching Notes:

- Emphasize "The LCM Method for Adding or Subtracting Rational Expressions" as given in the book.
- Remind students that they are working with an expression, not an equation. Many students will try to multiply every term in the expression by the LCM and then cancel out common factors.
- Remind students to watch the signs when they are performing subtraction.
- Many students will want to cancel terms. Emphasize reducing factors.
- Illustrate $\frac{1}{x}+\frac{1}{y}\neq\frac{2}{x+y}$ by showing $\frac{1}{2}+\frac{1}{3}\neq\frac{2}{5}$.
- Emphasize "Simplifying a Complex Rational Expression" as given in the book.

Answers:

1. ***(a)*** $\frac{y+8}{6(y+2)}$ ***(b)*** $-(x+9)$

2. ***(a)*** $\frac{1}{3x^6(x-7)}$ ***(b)*** $\frac{x-3}{x+8}$ ***(c)*** $\frac{6x}{(x-5)(x-7)}$

(d) $\left(\frac{x+2}{x-2}\right)^2$ ***(e)*** $\frac{(x-2)(x+2)}{5x-7}$ ***(f)*** $\frac{5-x}{x-8}$

(g) $\frac{5x^2+22x-45}{(x-9)(x+9)}$ ***(h*** $\frac{6x}{(x-7)(x-1)(x+5)}$ ***(i)*** $\frac{2(2x^2+5x-9)}{(x+3)(x-3)(x+4)}$

3. ***(a)*** $\frac{3x+1}{6x-1}$ ***(b)*** $\frac{-2x(x^2-6)}{(x+6)(6x^2+4x-3)}$

Mini-Lecture R.8
*n*th Roots; Rational Exponents

Learning Objectives:

1. Work with *n*th roots
2. Simplify radicals
3. Rationalize denominators and numerator
4. Simplify expressions with rational exponents

Examples:

1. Simplify each expression. Assume all variables are positive.

   $(a)\sqrt[3]{216}$ $(b)\sqrt[3]{-343}$ $(c)\sqrt[4]{\dfrac{x^{10}y^{10}}{x^2y^6}}$

   $(d)5\sqrt{7}+6\sqrt{7}$ $(e)\left(\sqrt{7}+5\right)\left(\sqrt{7}-6\right)$ $(f)3\sqrt[3]{2}-8\sqrt[3]{128}$
2. Rationalize the denominator.

   $(a)\dfrac{4}{\sqrt{6}}$ $(b)\dfrac{\sqrt{3}}{2-\sqrt{2}}$ $(c)\dfrac{\sqrt{6}}{6-\sqrt{3}}$
3. Rationalize the numerator.

   $\dfrac{\sqrt{x}-3}{x-9}$
4. Simplify each expression. Answers should have only positive exponents. Assume all variables are positive.

   $(a)64^{2/3}$ $(b)(-216)^{1/3}$ $(c)\left(\dfrac{64}{512}\right)^{2/3}$

   $(d)\left(x^4y\right)^{1/3}\left(xy^4\right)^{2/3}$ $(e)\ \dfrac{\left(-3y^{3/4}\right)^4}{y^{3/2}}$ $(f)\left(\dfrac{x^{1/3}}{x^{-1/6}y^{-2/3}}\right)^{-3}$

Teaching Notes:

- Make sure they understand how radicals can be combined. $2+3\sqrt{5}\neq 5\sqrt{5}$
- Remind students to see if a quotient can be reduced before rationalizing the expression. Example: $\dfrac{\sqrt{45}}{\sqrt{6}}$.
- Encourage students to learn powers up to and including: $2^6, 3^4, 4^3, 5^4$.
- Students should practice converting between the forms $\sqrt[n]{a}$ and $a^{1/n}$ until there is immediate recognition.

- Show an additional method of simplifying expressions such as $8^{2/3}$.
  $8^{2/3} = \left(2^3\right)^{2/3} = \left(2^{\cancel{3}}\right)^{2/\cancel{3}} = 2^2 = 4$
- Emphasize Examples 9 and 10 in preparation for calculus.

Answers:

1. ***(a)*** 6 ***(b)*** -7 ***(c)*** x^2y ***(d)*** $11\sqrt{7}$ ***(e)*** $-23-\sqrt{7}$ ***(f)*** $-29\sqrt[3]{2}$

2. ***(a)*** $\dfrac{2\sqrt{3}}{3}$ ***(b)*** $\dfrac{2\sqrt{3}+\sqrt{6}}{2}$ ***(c)*** $\dfrac{2\sqrt{6}+\sqrt{2}}{11}$

3. $\dfrac{1}{\sqrt{x}+3}$

4. ***(a)*** 16 ***(b)*** -6 ***(c)*** $\dfrac{1}{4}$ ***(d)*** x^2y^3 ***(e)*** $81y^{3/2}$ ***(f)*** $\dfrac{1}{x^{3/2}y^2}$