

Mini-Lesson Appendix B.1

Ellipses and Hyperbolas

Learning Objectives:

1. Find the equation of an ellipse given its foci and vertices.
2. Find the equation of a hyperbola given its foci and vertices.

Key Examples:

1. Graph $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Label the intercepts and foci.
2. Graph $4x^2 + y^2 = 4$. Label the intercepts and foci.
3. Write the standard-form equation of the ellipse with foci at $(0, 5)$ and $(0, -5)$ and vertices at $(0, 8)$ and $(0, -8)$.
4. Graph $x^2 - \frac{y^2}{9} = 1$. Label the intercepts and foci.
5. Graph $9y^2 - 4x^2 = 36$. Label the intercepts and foci.
6. Write the standard-form equation of the hyperbola with foci at $(-6, 0)$ and $(6, 0)$ and vertices at $(-5, 0)$ and $(5, 0)$.

Answers: 1) and 2) *See Additional Answers at end of Mini-Lessons.* 3) $\frac{x^2}{39} + \frac{y^2}{64} = 1$ 4) and 5) *See Additional*

Answers at end of Mini-Lectures. 6) $\frac{x^2}{25} - \frac{y^2}{11} = 1$

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Teaching Notes:

- Students may confuse the roles of a and b . In the standard forms of the equations of an ellipse and a hyperbola, not being sure which one goes with x and which one with y . Make sure they understand that for an ellipse, a is chosen as the larger number, while for an ellipse, a goes with the positive term, which may be less than, equal to, or greater than b .

ERROR PREVENTION

- A common student error is to confused the equations used for finding the value of c , which is used to find the coordinates of the foci of ellipses and hyperbolas: For an ellipse, $c^2 = a^2 - b^2$, while for a hyperbola, $c^2 = a^2 + b^2$. To help students keep this straight, show them with graphs that on an ellipse, the foci are *closer* to center than the vertices are, while in a hyperbola, the foci are *farther* from the center than vertices are.

Closure Questions:

- Is $\frac{x^2}{9} + \frac{y^2}{9} = 1$ the equation of an ellipse? Explain.

No; this equation can be rewritten as $x^2 + y^2 = 9$, which is the equation of a circle with center $(0, 0)$ and radius 9.

- Are the asymptotes of a hyperbola part of the graph? Explain why they are important and how they are used.

No; the asymptotes are not part of the hyperbola. They are lines that the hyperbola approaches but does not touch or cross. The asymptotes are used as guide lines for graphing a hyperbola.