Mini-Lesson 2.1

Perimeter, Circumference, and Area

Learning Objectives:

1. Find the perimeter or circumference of basic shapes.
2. Find the area of basic shapes.
3. Key vocabulary: *perimeter, circumference, area*

Key Examples:

1. The owner of an art framing store recommends using a frame that is 1.5 in. wide to frame a painting that is 15 in. by 18 in.
   a) What is the perimeter of the painting?
   b) What is the perimeter of the outside edge of the frame?

2. a) What is the exact circumference of a circle with diameter 22 cm?
   b) What is the circumference of a circle with radius 17 cm to the nearest tenth?

3. Graph quadrilateral *QRST* with vertices *Q*(2, 3), *R*(5, 3), *S*(5, −1), and *T*(−1, −1). What is the perimeter of *QRST*?

4. The dimensions of a bedroom are $12\frac{1}{2}$ ft by 14 ft. How much carpeting is needed to cover the floor? Give your answer in square feet.

5. The diameter of a circle is 26 cm.
   a) What is the area of the circle in terms of π?
   b) What is area of the circle using the approximation 3.14 for π?

1a) 66 in. 1b) 78 in. 2a) 22π cm 2b) 106.8 cm 3) *See Additional Answers at end of Mini-Lessons.*
4) 175 sq. ft 5a) 169π sq. cm 5b) 530.66 sq. cm

Mini-Lesson 2.1

Perimeter, Circumference, and Area

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Geometry
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eText, Section 2.1 (MML)

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Section 2.1

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Interactive Lecture Video
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Video Organizer
Section 2.1 (print)

Video Organizer
Section 2.1 (MML)

PowerPoints, Section 2.1

Teaching Notes:

- All formulas used for measurement in circles involve π. Students need to pay attention to whether directions ask them to give exact answers in terms of π. or provide decimal approximations.
- Students do not need to memorize the specific formulas for the perimeter of a rectangle, triangle, or square. In each case, they just need to add the lengths of the sides.

ERROR PREVENTION

- Students often make errors in using the appropriate units for measurements. Remind them that perimeter and circumference require (linear) units, while area requires square units.

Closure Questions:

- What are the differences between perimeter, circumference, and area?

  Perimeter and circumference measure the distance around a geometric figure. "Circumference" is the special name for the distance around a circle. Area measures the space enclosed by a geometric figure in a plane.

Mini-Lesson 2.2

Patterns and Inductive Reasoning

Learning Objectives:

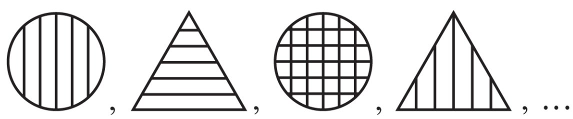
1. Use logic to understand patterns.
2. Understand and use inductive reasoning.
3. Key vocabulary: *inductive reasoning, induction, conjecture, counterexample*

Key Examples:

1. Look for a pattern in each list. Then use this pattern to predict the next number.

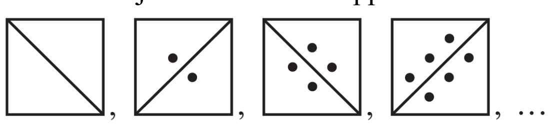
   a) 4, −8, 16, −32, 64, _______________ b) 6, 3, −1, −6, −12, _______________
   c) 2, 5, 7, 12, 19, _______________ d) 2, 6, 12, 20, 30, _______________

2. Notice two patterns in this sequence of figures. Use the patterns to draw the next figure in the sequence.



3. Study the list of squares. Use the pattern to answer the questions.

   a) Make a conjecture about the diagonal drawn in the 9th square.
   b) Make a conjecture about the number of dots on each side of the diagonal drawn in the 9th square.
   c) Make a conjecture about the appearance of the 9th square.
   d) Make a conjecture about the appearance of the 20th square.



Answers: 1a) −128; multiply by −2 1b) −19; subtract 3, then subtract 4, then subtract 5, and so on 1c) 31; each number starting with the third is the sum of the two previous numbers. 1d) 42; add 4, add 6, add 8, and so on 2) *See Additional Answers at end of Mini-Lessons.* 3a) In the 9th square, the diagonal goes downward from left to right. 3b) In the 9th square, there are 8 dots on each side of the diagonal. 3c) In the 9th square, the diagonal goes downward from left to right and there are 8 dots on each side of the diagonal. 3d) In the 20th square, the diagonal goes upward from left to right and there are 19 dots on each side of the diagonal.

Mini-Lesson 2.2

Patterns and Inductive Reasoning

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eText, Section 2.2 (MML)

Interactive Lecture Video Section 2.2

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 2.2 (print)

Video Organizer Section 2.2 (MML)

PowerPoints, Section 2.2

Teaching Notes:

- When looking for the pattern in a list of numbers, start by looking for a common difference or common ratio between each number that the preceding one. If that doesn't work, look for a more complicated pattern.

ERROR PREVENTION

- Students may "jump to conclusions" about the truth of a conjecture without looking at enough cases. If the conjecture is about numbers, encourage them to test the conjecture with a variety of kinds of numbers, such as both odd and even integers, or both positive and negative numbers.

Closure Questions:

- Does inductive reasoning guarantee that a conjecture is true?

  No; Inductive reasoning is based on observing patterns or specific examples. This does not prove that the conjecture is true in all possible cases.

- How many counterexamples are needed to prove that a conjecture is false? Explain.

  Only one counterexample is needed. A conjecture is only true if it is true in all possible cases, so finding one case in which it is not true proves that a conjecture is false.

Mini-Lesson 2.3

Conditional Statements

Learning Objectives:

1. Recognize conditional statements and their parts.
2. Write converses, inverses, and contrapositives of conditional statements.
3. Key vocabulary: *point, line, plane, lie on, collinear, coplanar, space, geometric figure, between, segment of line segment, ray, opposite rays, intersection*

Key Examples:

1. Identify the hypothesis (p) and the conclusion (q).
   a) If an animal is a horse, then the animal is a mammal.
   b) If n is an odd number, then n^2 is an odd number.

2. A hypothesis (p) and a conclusion (q) are given. Use them to write a conditional statement, $p \rightarrow q$.
   a) p: an angle measures 147° q: the angle is an obtuse angle
   b) p: a figure is a rectangle q: the figure is not a hexagon

3. Write the following statement in "if-then" form.
   Natural numbers are integers.

4. Determine whether each conditional statement is true or false.
   a) If two angles are complementary and congruent, then each angle measures 45°.
   b) If a month begins with the letter A, then the month has 30 days.

5. Write the (a) converse, (b) inverse, and (c) contrapositive of the given conditional statement.
   If two angles have the same measure, then the angles are congruent.

Answers: 1a) Hypothesis (p): an animal is a horse; Conclusion (q): the animal is a mammal 1b) Hypothesis (p): n is an odd number; Conclusion (q): n^2 is an odd number 2a) If an angle measures 147°, then the angle is an obtuse angle. 2b) If a figure is a rectangle, then the figure is not a hexagon. 3) If a number is a natural number, then the number is an integer. 4a) The statement is true. b) The statements is false. August begins with the letter A, but it has 31 days. 5a) If two angles are congruent, then the angles have the same measure. 5b) If two angles do not have the same measure, then the angles are not congruent. 5c) If two angles are not congruent, then the angles do not have the same measure.

Mini-Lesson 2.3

Conditional Statements

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eText, Section 2.3 (MML)

Interactive Lecture Video Section 2.3

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 2.3 (print)

Video Organizer Section 2.3 (MML)

PowerPoints, Section 2.3

Teaching Notes:

- To identify the hypothesis and conclusion in a conditional that does not contain the words *if* and *then,* tell students to first rewrite it in *if…then* form.
- Many students have trouble understanding that a conditional is true if the hypothesis and conclusion are both false. Explain this by with an example to show that if the hypothesis if false, no promise has been broken, so the conditional is true regardless of whether the conclusion is true or false.

ERROR PREVENTION

- When identifying the hypothesis and conclusion in a conditional statement, warn students that the hypothesis is not always written first.

Closure Questions:

- How do you form the contrapositive of a conditional statement?

  Negate the hypothesis and conclusion of the converse statement.

- Which pairs of a group of four related conditional statements are logically equivalent?

  The original conditional and its contrapositive are logically equivalent.

  The converse and the inverse are logically equivalent.

Mini-Lesson 2.4

Biconditional Statements and Definitions

Learning Objectives:

1. Write and understand biconditional statements.
2. Identify and understand good definitions.
3. Key vocabulary: *biconditional statement, good definition*

Key Examples:

1. Write the biconditional statement as a conditional statement and its converse.
   Two lines are parallel if and only if they are coplanar and do not intersect.

2. A true conditional statement is given:
   If a triangle has three congruent sides, it has three congruent angles.
   a) Write the converse of the conditional statement.
   b) Decide whether the converse statement is true of false.
   c) If the converse statement is true, write a true biconditional statement. If the converse statement is false, give a counterexample.

3. A true conditional statement is given:
   If $x = 11$, then $x^2 = 121$.
   a) Write the converse of the conditional statement.
   b) Decide whether the converse statement is true of false.
   c) If the converse statement is true, write a true biconditional statement. If the converse statement is false, give a counterexample.

4. **Multiple Choice** Which of the following is a good definition?
   A. A circle is a round figure.
   B. Two angles are congruent if they are both obtuse angles.
   C. A pound is a measure of weight equal to 16 ounces.
   D. Saturday is a day of the week that begins with the letter S.

5. Determine whether or not the following definition is a good one. To do so, attempt to write it as a true biconditional statement.
   Definition: A square is a figure with four congruent sides and four right angles.

Answers: 1) Conditional statement: If two lines are parallel, they are coplanar and do not intersect. Converse: If two lines are coplanar and do not intersect, they are parallel. 2a) If a triangle has three congruent angles, then it has three congruent sides. 2b) The converse statement is true. 2c) A triangle has three congruent sides if and only if it has three congruent angles. 3a) If $x^2 = 121$, then $x = 11$. 3b) The converse statement is false. 3c) $x = -11$ is a counterexample. 4) C is a good definition. 5) The definition is a good one.

Mini-Lesson 2.4

Biconditional Statement and Definitions

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eText, Section 2.4 (MML)

Interactive Lecture Video Section 2.4

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

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Video Organizer Section 2.4 (MML)

PowerPoints, Section 2.4

Teaching Notes:

- Emphasize that a good definition can be written as a true biconditional. This is important because we need to use precise definitions for the many vocabulary words we use in geometry.

ERROR PREVENTION

- Often students make the error of assuming that because a conditional is true, its converse must also be true. Remind them to look for counterexamples.

Closure Questions:

- What is the key phrase for a biconditional statement?

  if and only if

- What must be true for a biconditional statement to be true?

  Both a conditional and its converse must be true.

Mini-Lesson 2.5

Deductive Reasoning

Learning Objectives:

1. Review conditional, converse, inverse, and contrapositive statements.
2. Understand and use two laws of deductive reasoning: the Law of Detachment and the Law of Syllogism.
3. Key Vocabulary: *deductive reasoning, Law of Detachment, Law of Syllogism*

Key Examples:

1. Use the given conditional statement to write its a) converse, b) inverse, and c) contrapositive statements. Then write each statement in symbols. ($p \rightarrow q$)
   If a number is an integer, then it is a rational number.

2. Use the Law of Detachment to make a true conclusion. (Assume that the first statement $p \rightarrow q$ is true.)
   If the temperature is below 32°F, water will freeze.
   The temperature is 28°F today.

3. Determine whether each reasoning is valid using the Law of Detachment. (Assume that the first statement $p \rightarrow q$ is true.)
   a) If a student's average test score in Geometry is 80–89.9%, he or she will receive a B in the course. Eli's average test score in Geometry was 84.5%, so he will receive a B in the course.
   b) If a figure is a square, then it has four right angles. Figure *EFGH* has four right angles, so the figure is a square.

4. Use the Law of Syllogism to form a true conclusion ($p \rightarrow r$).
   Given: If the sum of the digits of a natural number is divisible by 9, then the number is divisible by 9.
   If a natural number is divisible by 9, then the number is divisible by 3.

5. Decide what you can conclude from the true conditional statements given, and note whether your reasoning involves the Law of Detachment or the Law of Syllogism.
   a) **Given:** All birds have feathers.
   Parrots are birds.
   b) **Given:** All natural numbers that are divisible by 8 are divisible by 4.
   All natural numbers that are divisible by 4 are divisible by 2.

Answers: 1a) If a number is a rational number, then it is an integer; $q \rightarrow p$ 1b) If a number is not an integer, then it is not a rational number; $\sim p \rightarrow \sim q$ 1c) If a number is not a rational number, then it is not an integer; $\sim q \rightarrow \sim p$
2) Water will freeze today. 3a) valid 3b) not valid 4) If the sum of the digits of a natural number is divisible by 9, the number is divisible by 3. 5a) Parrots have feathers; Law of Detachment 5b) All natural numbers that are divisible by 8 are divisible by 2; Law of Syllogism

Mini-Lesson 2.5

Deductive Reasoning

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eText, Section 2.5 (MML)

Interactive Lecture Video Section 2.5

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 2.5 (print)

Video Organizer Section 2.5 (MML)

PowerPoints, Section 2.5

Teaching Notes:

- We use deductive reasoning to prove theorems.
- The Law of Detachment says that for a true conditional, if the hypothesis is true, the conclusion must be true. It does not say anything about the situation in which the hypothesis is false.

ERROR PREVENTION

- If students have trouble with the Law of Detachment, have them identify the hypothesis and conclusion in each of the three conditional statements.

Closure Questions:

- Why can't a proof be based on inductive reasoning?

  Inductive reasoning is based on specific examples, so it may lead to a conclusion that is not true because it is not true in all cases.

- What is a key difference between the Law of Detachment and the Law of Syllogism?

  The Law of Detachment involves two statements, while the Law of Syllogism involves three statements.

Mini-Lesson 2.6

Reviewing Properties of Equality and Writing Two-Column Proofs

Learning Objectives:

1. Use properties of equality to justify reasons for steps.
2. Write a two-column proof.
3. Key vocabulary: *reflexive property, symmetric property, transitive property, substitution property, proof, two-column proof*

Key Examples:

1. Solve $7x+18=81$. Give a reason to justify each statement.

2. Solve $15x-2(30+4x)=22x$. Give a reason to justify each statement.

3. Fill in each blank with the reason to justify the statement.

   a)

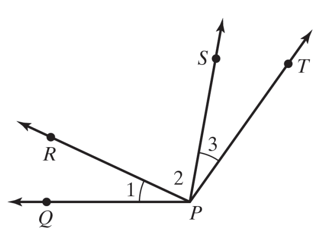
Statements	**Reasons**
$m\angle R = m\angle S$	Given
$m\angle S = m\angle T$	Given
$m\angle R = m\angle T$	____________________

   b)

Statements	**Reasons**
$AB + CD = 10$	Given
$AB = EF$	Given
$EF + CD = 10$	____________________

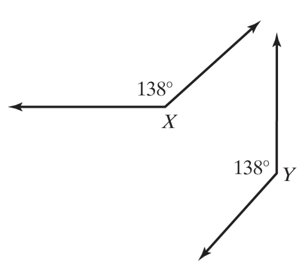
4. Write a two-column proof.

   Given: $m\angle 1 = m\angle 3$
   Prove: $m\angle QPS = m\angle TPR$



5. Write a two-column proof.

   Given: $m\angle X = 138°$, $m\angle Y = 138°$
   Prove: $\angle X \cong \angle Y$



Answers: 1) and 2) *See Additional Answers at end of Mini-Lessons.* 3a) Transitive Property 3b) Substitution Property 4) and 5) *See Additional Answers at end of Mini-Lessons.*

Mini-Lesson 2.6

Reviewing Properties of Equality and Writing Two-Column Proofs

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eText, Section 2.6 (MML)

Interactive Lecture Video Section 2.6

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 2.6 (print)

Video Organizer Section 2.6 (MML)

PowerPoints, Section 2.6

Teaching Notes:

- Some students may need to review the steps for solving a linear equation.
- Review the relationship between equality and congruence for segments and angles.

ERROR PREVENTION

- Remind students of the correct usage of the equal sign vs. the congruence symbol.

Closure Questions:

- What is an easy way to distinguish between the reflexive property, the symmetric property, and the transitive property?

  The reflexive property involves one number, the symmetric property involves two numbers, and the transitive property involves three numbers.

- Which law of deductive reasoning is similar to the transitive property?

  the Law of Syllogism

Mini-Lesson 2.7
Proving Theorems About Angles

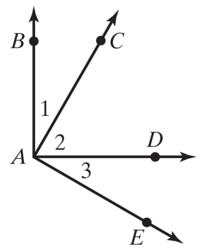
Learning Objectives:

1. Prove and use theorems about angles.
2. Key vocabulary: *paragraph proof*

Key Examples:

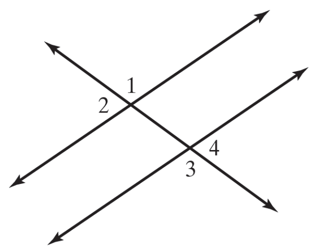
1. Write a two-column proof.

   Given: $\overrightarrow{AB} \perp \overrightarrow{AD}$ and $\overrightarrow{AC} \perp \overrightarrow{AE}$
   Prove: $\angle 1 \cong \angle 3$



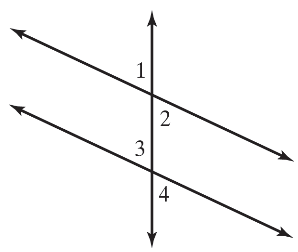
2. Write a two-column proof.

   Given: $\angle 1 \cong \angle 3$
   Prove: $\angle 2 \cong \angle 4$

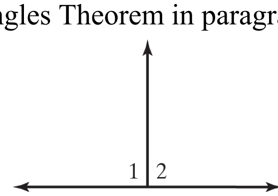


3. Write a two-column proof.

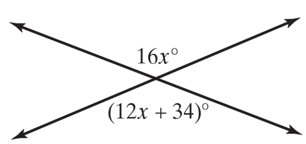
   Given: $\angle 2 \cong \angle 3$
   Prove: $\angle 1 \cong \angle 4$



4. Write a proof of the Equal Supplementary Angles Theorem in paragraph form.
   Given: $m\angle 1 = m\angle 2$ and $m\angle 1 + m\angle 2 = 180°$
   Prove: $\angle 1$ and $\angle 2$ are right angles.



5. Find the value of *x*.



Answers:
1)–4) *See Additional Answers at the end of the Mini-Lectures.* 5) 8.5

Mini-Lesson 2.7

Proving Theorems About Angles

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eText, Section 2.7 (MML)

Interactive Lecture Video Section 2.7

Interactive Lecture Video Objective 1

Video Organizer Section 2.7 (print)

Video Organizer Section 2.7 (MML)

PowerPoints, Section 2.7

Teaching Notes:

- To help students learn the theorems in this lesson, ask them to write out each theorem in words without looking at the textbook.
- The statements and proofs of several of the theorems in this lesson involve adding angle measures. Remind students that we are adding angle *measures,* which are numbers, not adding angles.

ERROR PREVENTION

- When writing a paragraph proof, students often leave out some of the reasons. To make sure that every statement is supported by a reason, ask them to rewrite these proofs in two-column form.

Closure Questions:

- How can you translate a conditional statement into the "Given" and "Prove" for a proof?

  The hypothesis, or "if" statement, becomes the "Given" and the conclusion or "then" statement becomes the "Prove."

- What kinds of Reasons are used in the two-column proofs in this lesson?

  Given, definitions, properties of equality, postulates, previously proved theorems,