Mini-Lesson 4.1

Types of Triangles

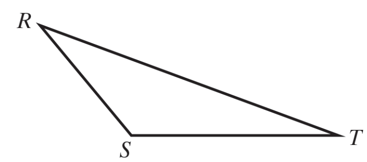
Learning Objectives:

1. Use the vocabulary of triangles.
2. Classify triangles by angles and sides.
3. Find angle measures of triangles.
4. Key vocabulary: *triangle, vertex, sides of a triangle, adjacent sides, opposite side and angle, included side and angle, acute triangle, obtuse triangle, equiangular triangle, right triangle, scalene triangle, isosceles triangle, equilateral triangle, interior angle, exterior angle, corollary*

Key Examples:

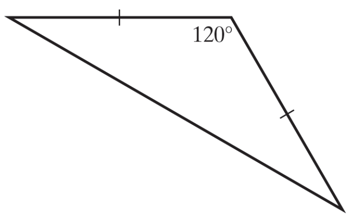
1. Given ΔTRS :

   a) Which angle is opposite $\overline{RT}$?
   b) Which side is opposite $\angle T$?
   c) Which side is included between $\angle S$ and $\angle T$?
   d) Which angle is included between $\overline{RS}$ and $\overline{RT}$?

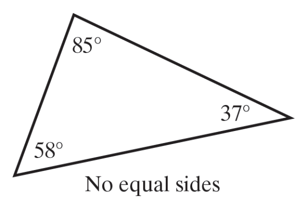


2. Classify each triangle based on its sides and angles. Use the most specific name.

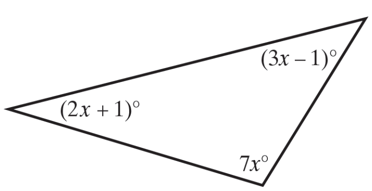
   a)



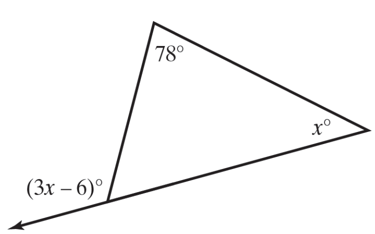
   b)



3. Use the Triangle-Angle Sum Theorem to find the measure of each angle in the given triangle.



5. Use the Exterior Angle of a Triangle Corollary to find the measure of the exterior and the nonadjacent angle shown.



Answers: 1a) $\angle S$ 1b) $\overline{RS}$ 1c) $\overline{ST}$ 1d) $\angle R$ 2a) obtuse triangle; isosceles triangle; an obtuse isosceles triangle 2b) acute triangle; scalene triangle; an acute scalene triangle 3) $7x° = 105°$, $(2x + 1)° = 31°$, $(3x - 1)° = 44°$ 4) $x° = 42°$; $(3x - 6)° = 120°$

Mini-Lesson 4.1

Types of Triangles

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 4.1 (MML)

Interactive Lecture Video
Section 4.1

Interactive Lecture Video
Objective 1

Interactive Lecture Video
Objective 2

Interactive Lecture Video
Objective 3

Video Organizer
Section 4.1 (print)

Video Organizer
Section 4.1 (MML)

PowerPoints, Section 4.1

Teaching Notes:

- Emphasize that the Triangle-Sum Theorem is only true because of the Parallel Postulate. This is what distinguishes Euclidean geometry from other ("non-Euclidean") geometries. This concept can best be illustrated with spherical geometry by bringing in a globe and demonstrating that the sum of the angles in a triangle drawn on the surface of a sphere is *greater than* 180°.

ERROR PREVENTION

- Students may confuse the two different meanings of *leg* in a right triangle and an isosceles triangle. Point out that in an isosceles right triangle, both meanings described the same pair of sides.

Closure Questions:

- Which of the following is *not* possible?

  a scalene acute triangle
  an equilateral right triangle
  an isosceles obtuse triangle
  a scalene right triangle
  an isosceles right triangle

  an equilateral right triangle

- Why do we usually show only one of the two exterior angles at each vertex of a triangle?

  The two exterior angles at the same vertex are a pair of vertical angles, so they are congruent.

- Can an interior angle and an exterior angle of a triangle with the same vertex ever be congruent? If so, when?

  An interior angle and exterior angle with the same vertex form a linear pair, so they are supplementary. Two supplementary angles are congruent if and only if they are right angles. So, this occurs only at the right-angle vertex of a right triangle.

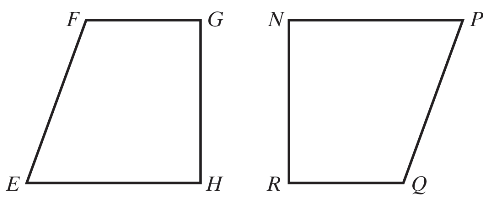
Mini-Lesson 4.2
Congruent Figures

Learning Objectives:

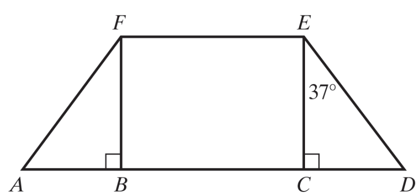
1. Identify corresponding parts in congruent triangles.
2. Prove triangles are congruent.
3. Key vocabulary: *congruent figures, corresponding sides, corresponding angles*

Key Examples:

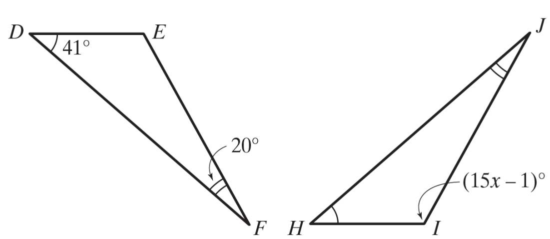
1. For the two figures, we are given that $EFGH \cong PQRN$, Name the corresponding angles and sides.



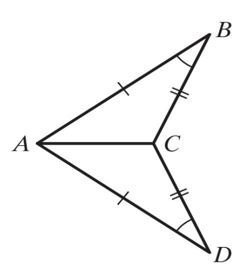
2. The figure represents a three-panel window made up of a rectangle and two congruent triangles. Given that $\Delta AFB \cong \Delta DEC$, find $m\angle A$.



3. Find the value of *x*.



4. **Given:** $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{DC}$, $\angle B \cong \angle D$,
$\overline{AC}$ bisects $\angle BAD$.
Prove: $\Delta ABC \cong \Delta ADC$



Answers: 1) Angles: $\angle E \cong \angle P$, $\angle F \cong \angle Q$, $\angle G \cong \angle R$, $\angle H \cong \angle N$; Sides: $\overline{EF} \cong \overline{PQ}$, $\overline{FG} \cong \overline{QR}$, $\overline{GH} \cong \overline{RN}$, $\overline{HE} \cong \overline{NP}$ 2) 53° 3) $x = 8$ 4) *See Additional Answers at end of Mini-Lessons.*

Mini-Lesson 4.2

Congruent Figures

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 4.2 (MML)

Interactive Lecture Video Section 4.2

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 4.2 (print)

Video Organizer Section 4.2 (MML)

PowerPoints, Section 4.2

Teaching Notes:

- Emphasize that a pair of congruent triangles has 6 pairs of corresponding parts: 3 pairs of sides and 3 pairs of angles.
- When writing a congruence statement, emphasize that what matters is that vertices are listed in corresponding positions for the two congruent figures, not the order in which the vertices are listed in the first figure. However, the vertices should be listed consecutively to match the figure; they do not need to be listed alphabetically.

ERROR PREVENTION

- When writing congruence statements, students often make errors in writing corresponding vertices in corresponding positions. Provide as much extra practice as they need to learn to do this accurately.

Closure Questions:

- If two pentagons (5-sided figures) are congruent, how many pairs of corresponding congruent parts will there be? Describe them.

  There will be 10 pairs of corresponding parts: 5 pairs of corresponding congruent angles and 5 pairs of corresponding congruent angles.

Mini-Lesson 4.3

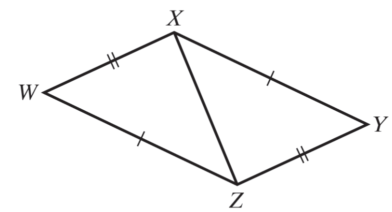
Congruent Triangles by SSS and SAS

Learning Objectives:

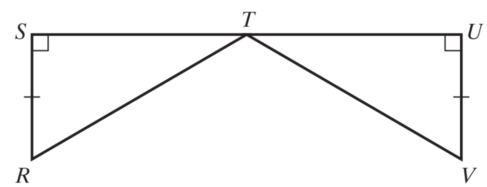
1. Prove two triangles are congruent using the SSS and SAS postulates.

Key Examples:

1. **Given:** $\overline{WX} \cong \overline{YZ}$, $\overline{WZ} \cong \overline{YX}$
   Prove: $\Delta WXZ \cong \Delta YZX$

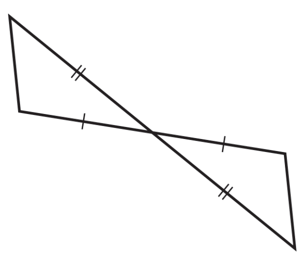


2. **Given:** $\overline{RS} \cong \overline{VU}$, $\overline{RS} \perp \overline{ST}$, $\overline{VU} \perp \overline{UT}$,
   T is the midpoint of $\overline{SU}$.
   Prove: $\Delta RST \cong \Delta VUT$

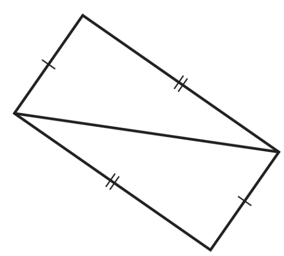


3. Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove by SSS or SAS, then write "not enough information" and explain why.

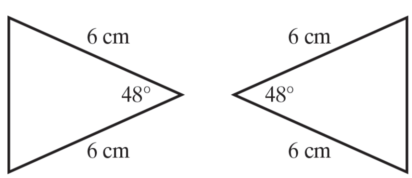
a)



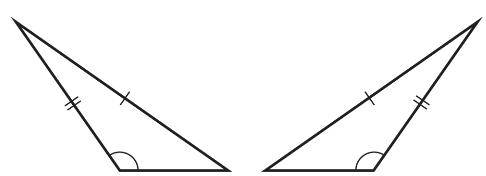
b)



c)



d)



Answers: 1) and 2) *See Additional Answers at end of Mini-Lessons.* 3a) SAS 3b) SSS 3c) SAS 3d) not enough information; two pairs of corresponding sides are congruent, but the angles marked congruent are not the included angles.

Mini-Lesson 4.3

Congruent Triangles by SSS and SAS

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Geometry
Multimedia Lesson Resources

eText, Section 4.3 (MML)

Interactive Lecture Video Section 4.3

Interactive Lecture Video Objective 1

Video Organizer Section 4.3 (print)

Video Organizer Section 4.3 (MML)

PowerPoints, Section 4.3

Teaching Notes:

- Remind students that whenever they write a triangle congruence statement, they must write corresponding vertices in corresponding positions.
- For SAS, emphasize that we write A in the middle to show that the angles used in the two triangles must be *included* between the two sides. Show some examples to show that SSA (two sides and a non-included angle) does *not* guarantee congruent triangles, as in some cases two different triangles can be drawn with the same given SSA parts.

ERROR PREVENTION

- A common error when using SAS is to use non-included angles. Provide students with extra figures in which they must identify the angle included between two specified sides.

Closure Questions:

- What are two conditions that guarantee that two triangles are congruent given three pairs of congruent parts? Name each condition and state it in words.

  SSS Postulate: If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

  SAS Postulate: If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

Mini-Lesson 4.4

Congruent Triangles by ASA and AAS

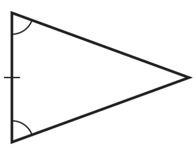
Learning Objectives:

1. Prove two triangles are congruent using the ASA Postulate and the AAS Theorem.
2. Identify when to use SSS, SAS, ASA, or AAS to prove triangles congruent.

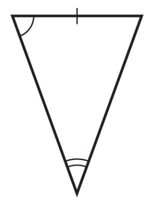
Key Examples:

1. **Multiple Choice** Choose two triangles that are congruent by the ASA Postulate. Explain why.

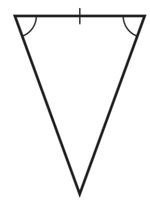
A.



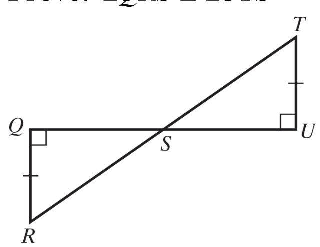
B.



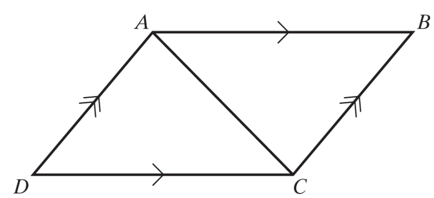
C.



2. **Given:** $\overline{QR} \cong \overline{UT}$, $\overline{QR} \perp \overline{QS}$, $\overline{UT} \perp \overline{US}$
Prove: $\triangle QRS \cong \triangle UTS$

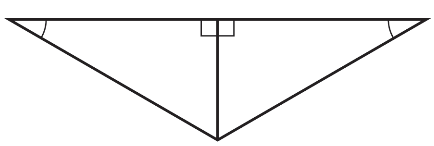


3. **Given:** $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$,
Prove: $\triangle ADC \cong \triangle BCA$

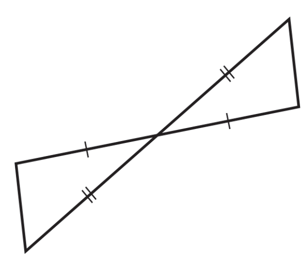


4. Use SSS, SAS, ASA, or AAS to identify the postulate or theorem that immediately confirms the congruence for each pair of triangles.

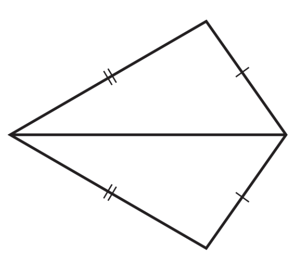
a)



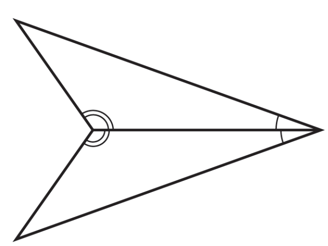
b)



c)



d)



Answers: 1) A and C are congruent by ASA because the sides marked congruent in these triangles are the included sides of the two congruent angles. 2) and 3) *See Additional Answers at end of Mini-Lessons.* 4a) AAS 4b) SAS 4c) SSS 4d) ASA

Mini-Lesson 4.4

Congruent Triangles by ASA and AAS

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Geometry
Multimedia Lesson Resources

eText, Section 4.4 (MML)

Interactive Lecture Video Section 4.4

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 4.4 (print)

Video Organizer Section 4.4 (MML)

PowerPoints, Section 4.4

Teaching Notes:

- For with SAS, emphasize that we write S in the middle in ASA to show that the sides used in the two triangles must be *included* between the two angles. If the angles used are *not* included, the triangles will still be congruent, but by a different congruence condition: the AAS Theorem.
- Have students use their protractors to sketch several triangles with the same given angle measures. This should convince them that AAA is *not* a congruence condition.

ERROR PREVENTION

- A common error is not to correctly distinguish between cases of ASA and AAS. Provide students with extra figures in which state which of they must identify the side included between two specified sides.

Closure Questions:

- Why is the AAS congruence condition a theorem, while SSS, SAS, and ASA are postulates?

  We are not able to prove the SSS, SAS, and ASA conditions, so must assume that they are true, making them postulates. But, we can prove the AAS condition using the ASA Postulate and the Third Angle Theorem, so AAS is a theorem.
- Which combinations of three congruent sides and/or angles guarantee that two triangles will be congruent? Which combinations do not guarantee congruence?

  SSS, SAS, ASA, and AAS guarantee congruence. SSA and AAA do not.

Mini-Lesson 4.5

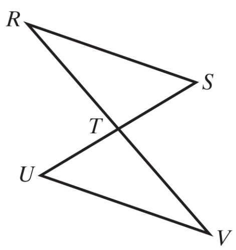
Proofs Using Congruent Triangles

Learning Objectives:

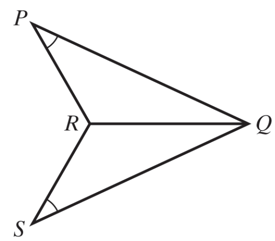
1. Use triangle congruence and corresponding parts of congruent triangles to prove that parts of two triangles are congruent.
2. Prove two triangles are congruent using other congruent triangles.
3. Key vocabulary: *cpoctac*

Key Examples:

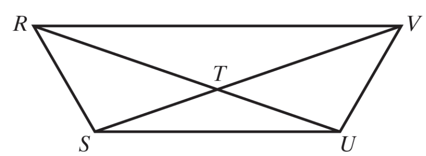
1. **Given:** T is the midpoint of $\overline{RV}$.
   T is the midpoint of $\overline{SU}$.
   Prove: $\overline{RS} \cong \overline{VU}$



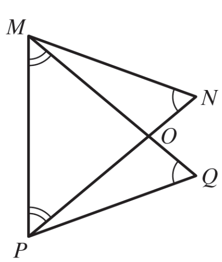
2. **Given:** $\overline{QR}$ bisects $\angle PQS$.
   $\angle P \cong \angle S$
   Prove: $\angle PRQ \cong \angle SRQ$



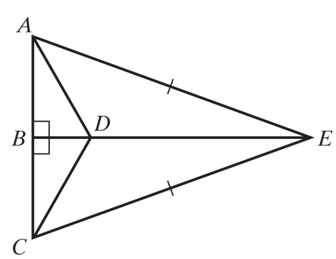
3. a) What is the common side in ΔRSV and ΔVUR?
   b) What is the common side in ΔRSU and ΔVUS?



4. **Given:** $\angle N \cong \angle Q$, $\angle NPM \cong \angle QMP$
   Prove: $\overline{MN} \cong \overline{PQ}$



5. **Given:** $\overline{AC} \perp \overline{BE}$, $\overline{AE} \cong \overline{CE}$,
   B is the midpoint of $\overline{AC}$.
   Prove: $\Delta ADE \cong \Delta CDE$



Answers: 1) and 2) *See Additional Answers at end of Mini-Lessons.* 3a) $\overline{RV}$ 3b) $\overline{SU}$ 4) and 5) *See Additional Answers at end of Mini-Lessons.*

Mini-Lesson 4.5

Proofs Using Congruent Triangles

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 4.2 (MML)

Interactive Lecture Video Section 4.2

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 4.2 (print)

Video Organizer Section 4.2 (MML)

PowerPoints, Section 4.2

Teaching Notes:

- To introduce cpoctac, review a few proofs from the examples or Exercise Set from Section 4.4. After the last step showing the triangle congruence, ask students which pairs of parts of the two triangles that were not used in the proof they now know are congruent.

ERROR PREVENTION

- Students often have trouble visualizing or keeping track of overlapping triangles. The textbook suggests that they draw the two triangles separately. Another thing that may help is to outline the two triangles in different colors. You can demonstrate this in class and then students can try it on their own with colored pencils.

Closure Questions:

- What does "cpoctac" mean? How is it used in triangle congruence parts?

  *It means "**C**ongruent **P**arts **O**f **C**ongruent **T**riangles **A**re Congruent." Once you have proved that two triangles are congruent by using a combination of three corresponding parts (SSS, SAS, ASA, or AAS), you can use cpoctac to conclude that any of the three remaining pairs of corresponding parts are congruent.*

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Mini-Lesson 4.6

Isosceles, Equilateral, and Right Triangles

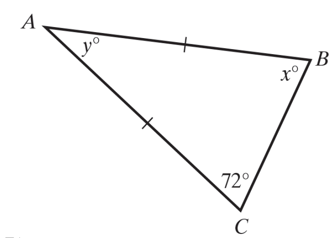
Learning Objectives:

1. Use properties of isosceles and equilateral triangles.
2. Use properties of right triangles.
3. Key vocabulary: *legs of an isosceles triangle, base of an isosceles triangle, vertex angle of an isosceles triangle, bases angles of an isosceles triangle, hypotenuse, legs of a right triangle*

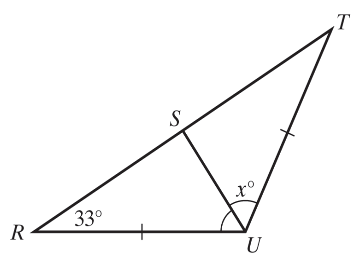
Key Examples:

1. Write a biconditional statement that combines the Isosceles Base Angles Theorem and its converse.

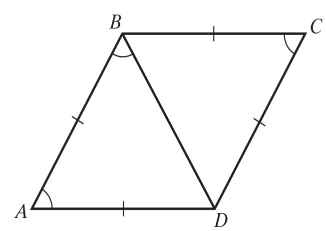
2. Use the figure and markings to find the values of x and y.



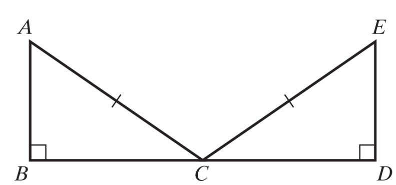
3. Use the figure to find the value of x.



4. Use the figure to find the measures of the four angles in quadrilateral *ABCD.*



5. **Given:** $\overline{AB} \perp \overline{BC}$, $\overline{ED} \perp \overline{DC}$, $\overline{AC} \cong \overline{EC}$
   C is the midpoint of $\overline{BD}$.
   Prove: $\Delta ABC \cong \Delta EDC$



Answers: 1) Two sides of a triangle are congruent if and only if the angles opposite those sides are congruent. 2) $x = 72$, $y = 36$ 3) $x = 57$ 4) $m\angle A = 60°$, $m\angle ABC = 120°$, $m\angle C = 60°$, $m\angle CDA = 120°$ 5) *See Additional Answers at end of Mini-Lessons.*

Mini-Lesson 4.6

Isosceles, Equilateral, and Right Triangles

MyMathLab®

***Geometry* Multimedia Lesson Resources**

eText, Section 4.6 (MML)

Interactive Lecture Video Section 4.6

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 4.6 (print)

Video Organizer Section 4.6 (MML)

PowerPoints, Section 4.6

Teaching Notes:

- Emphasize the vocabulary of isosceles triangles and correctly identifying the *base, legs, base angles,* and *vertex angle.* Explain the difference between the use of “base” in an isosceles triangle vs. when finding the area of a triangle, and the difference between the *legs* of an isosceles triangle and the legs of a right angle.
- Students may be confused as to why all equilateral triangles are isosceles triangles. Remind them of the definition of an isosceles triangle: a triangle with *at least* two congruent sides.

ERROR PREVENTION

- Students often thing that the base of a triangle must be the “bottom side.” Give them several isosceles triangles with a variety of orientations and ask them to identify the base in each one.

Closure Questions:

- How can the Isosceles Base Angles Theorem and its converse be combined into one biconditional statement?

  Two sides of a triangle are congruent if and only if the angles opposite those two sides are congruent.

- Every equilateral triangle is an isosceles triangle, but the terms *base, legs, base angles,* and *vertex angle* are rarely used when talking about equilateral triangles. Why do you think this is so?

  Because all three sides of an equilateral triangle are congruent, any pair of sides could be considered as the legs and the third side as the base. Because all three angles of an equilateral triangle are congruent, any pairs of sides could be considered as the base angles and the third angle as the vertex angle. So, there is no reason to identify sides and angles of an equilateral triangle with these special names.