Mini-Lesson 5.1

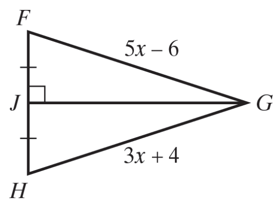
Perpendicular and Angle Bisectors

Learning Objectives:

1. Use perpendicular bisectors to solve problems.
2. Use angle bisectors to solve problems.
3. Key vocabulary: *equidistant, distance from a point to a line*

Key Examples:

1. Use the given figure to find the length of $\overline{FG}$.



2. An entrepreneur wishes to open a convenience store in a town that already has two such stores. Based on the information in the figure, where should the store be located so that it is equidistant from the other two stores, and still be as far from them as possible?

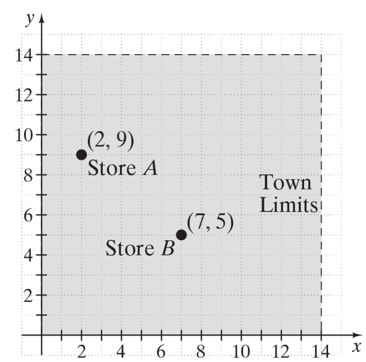
   a) Explain the process for finding the location.

   b) Find the coordinate of the midpoint of $\overline{AB}$.

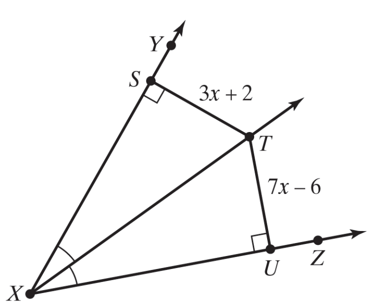
   c) Find the slope of $\overline{AB}$, and of a line perpendicular to $\overline{AB}$.

   d) Use the perpendicular slope in part c) to draw the perpendicular bisector of $\overline{AB}$.

   e) Approximate the coordinates for the new store. (Use whole numbers coordinates within the town limits.)



3. Use the given figure and find the length of $\overline{TU}$.



Answers: 1) 19 units 2a) All points along the perpendicular bisector of *A* and *B* are equidistant from those two points. By approximating the coordinates of the intersection of the perpendicular bisector and the town limits, the ideal location for the new store may be found. 2b) (4.5, 7) 2c) slope of $\overline{AB}$ = –0.8; slope of perpendicular (m) = 1.25 2d) *See Additional Answers at end of Mini-Lessons*. 2e) (10, 13) or (10,14) 3) 8 units

Mini-Lesson 5.1

Perpendicular and Angle Bisectors

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Geometry
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eText, Section 5.1 (MML)

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Section 5.1

Interactive Lecture Video
Objective 1

Interactive Lecture Video
Objective 2

Video Organizer
Section 5.1 (print)

Video Organizer
Section 5.1 (MML)

PowerPoints, Section 5.1

Teaching Notes:

- Start this lesson with a quick review of the congruence postulates and theorems from Chapter 4, some of which are used in the proofs of the theorems in this section.
- Remind students of the meaning of "cpoctac" and how it is used in proofs.

ERROR PREVENTION

- Because an infinite number of segments of different lengths can be drawn to a line from a point not on the line, students may be confused by the concept of "the distance from a point to a line." Explain that we *define* the distance from a point to a line to be a perpendicular distance so that everybody will agree on the same distance and also because it is the length of the *shortest* segment that can be drawn to a line from a point not on the line.

Closure Questions:

- What does it mean when we say that a point is equidistant from two lines?

  It means that the lengths of the perpendicular segments from the point to the two lines are equal.

Mini-Lesson 5.2
Bisectors of a Triangle

Learning Objectives:

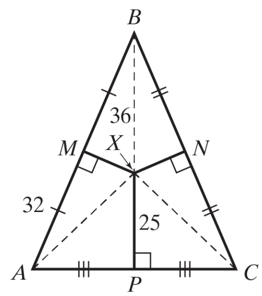
1. Use properties of the perpendicular bisectors of the sides of a triangle, including the circumcenter.
2. Use properties of the angle bisectors of the angles of a triangle, including the incenter.
3. Key vocabulary: *concurrent, point of concurrency, circumcenter of a triangle, circumscribed about, incenter of a triangle, inscribed in*

Key Examples:

1. The circumcenter of ΔABC is point X. Fill in the blanks.

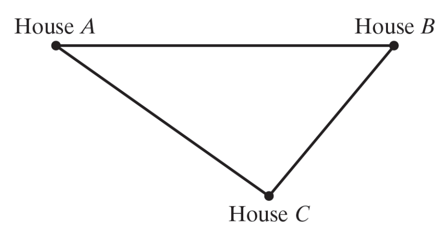
   a) AX = _______ = ________
   (Use segment distances here.)

   b) AX = _______ units

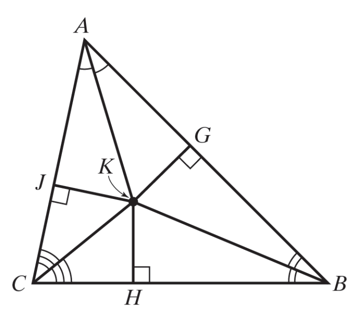


2. What are the coordinates of the circumcenter of the triangle with vertices $F(0, 1)$, $G(3, 1)$, and $H(3, 5)$?

3. A mailbox for the neighborhood is to be located equidistant from three houses: A, B, and C. Sketch the location of the mailbox.



4. $KJ = 3x + 12$ and $KH = 6x - 15$. What is KG?



Answers: 1a) *BX*, *CX* 1b) 36 2) (1.5, 3) 3) *See Additional Answers at end of Mini-Lessons*. 4) 39 units

Mini-Lesson 5.2

Bisectors of a Triangle

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eText, Section5.2 (MML)

Interactive Lecture Video Section 5.2

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 5.2 (print)

Video Organizer Section 5.2 (MML)

PowerPoints, Section 5.2

Teaching Notes:

- Make sure that students understand and can distinguish between the concepts of the perpendicular bisectors of the *sides* of a triangle and the bisectors of the *angles* of a triangle before introducing the concepts of circumscribed and inscribed circles.

ERROR PREVENTION

- Students are likely to confuse the *cimcumscribed circle* with the *inscribed circle,* and the *circumcenter* with the *incenter.* Tell them that the prefix "circum" means "around" (the circumference is the distance *around* a circle) and that they should always think about whether the circle is *around* the triangle (circumcircle/circumcenter) or the circle is *inside* the triangle (inscribed circle/incenter). Give students a variety of triangles and ask them to sketch (rather than construct) both of these circles for the each triangle to reinforce these concepts.

Closure Questions:

- How is the location of the circumcenter of a triangle related to the type of triangle (acute, right, or obtuse)?

  The circumcenter of an acute triangle is inside the triangle, the circumcenter of a right triangle is on the triangle, and the circumcenter of an obtuse triangle is outside the triangle.

- In what type of triangle are circumcenter and incenter at the same point? In this case, are the circumcenter and incenter at the center of the same circle? Explain.

  In an equilateral triangle, the circumcenter and incenter are the same point, but the circumcircle and incircle are not the same circles. These are two circles with the same center, but the circumcircle goes around the triangle, while the incircle is inside it.

Mini-Lesson 5.3

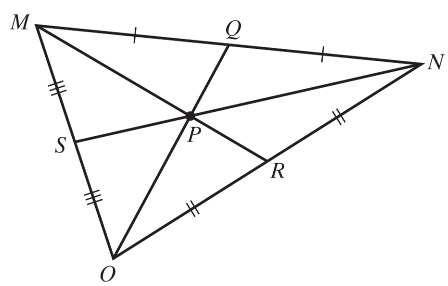
Medians and Altitudes of a Triangle

Learning Objectives:

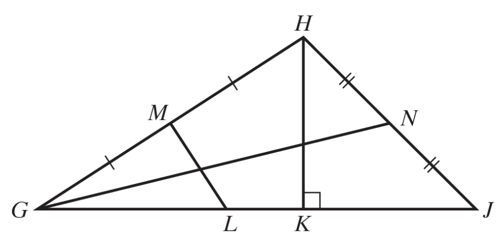
1. Use properties of the medians of a triangle.
2. Use properties of the altitudes of a triangle.
3. Key vocabulary: *median of a triangle, centroid of a triangle, altitude of a triangle, orthocenter of a triangle*

Key Examples:

1. In the diagram, NP = 16 units. Find NS.



2. For ΔGHJ, is each segment a *median*, an *altitude*, or *neither*? Explain.
   a) $\overline{GN}$
   b) $\overline{HK}$
   c) $\overline{ML}$



3. ΔRST has vertices R (3, 0), S (7, 5), and T (7, 0). What are the coordinates of the orthocenter of ΔRST?

Answers: 1) 24 units 2a) median; It connects a vertex of ΔGHJ and the midpoint of the opposite side. 2b) altitude; It is the segment from vertex H to the line containing the opposite side $\overline{GJ}$. 2c) neither; M is a midpoint of ΔGHJ, but L is not a vertex of ΔGHJ. 3) (7, 0)

Mini-Lesson 5.3

Medians and Altitudes of a Triangle

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eText, Section 5.3 (MML)

Interactive Lecture Video Section 5.3

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 5.3 (print)

Video Organizer Section 5.3 (MML)

PowerPoints, Section 5.3

Teaching Notes:

- Make sure that students understand and can distinguish between the concepts of the *medians* and *altitudes* of a triangle before introducing the concepts of *centroid* and *orthocenter.* Give students an acute triangle and ask them to sketch (not construct) the three medians of the triangle, and then, on a copy of the same triangle, sketch the three altitudes. Repeat with a right triangle and an obtuse triangle. (In each case, use a scalene triangle.)
- Because the circumcircle is the center of the circumscribed circle and the incenter is the center of the inscribed circle, students might logically expect the orthocenter to be the center of the "orthocircle." Explain that there is no such circle, and that the term "center" is used for all four points of concurrency, even though the centroid and orthocenter are not the centers of circles.

ERROR PREVENTION

- Students are very likely to confuse the four centers of a triangle (points of concurrency). The Helpful Hint on p.224 is very helpful because it illustrates all four of these side-by-side in the same acute triangle. Ask students to repeat this process with a right triangle and an obtuse triangle.

Closure Questions:

- How is the location of the orthocenter of a triangle related to the type of triangle (acute, right, or obtuse)?

  The orthocenter of an acute triangle is inside the triangle, the orthocenter of a right triangle is on the triangle, and the orthocenter of an obtuse triangle is outside the triangle.

- In what type of triangle are the three medians the same segments as the three perpendicular bisectors, the three angle bisectors, and the three altitudes? What does this tell you about the four types of centers for this type of triangle?

  In an equilateral triangle, the medians, perpendicular bisectors, angle bisectors, and altitudes are the same three segments, so the centroid, circumcenter, incenter, and orthocenter are all the same point.

Mini-Lesson 5.4

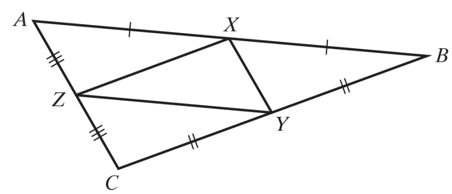
Midsegments of Triangles

Learning Objectives:

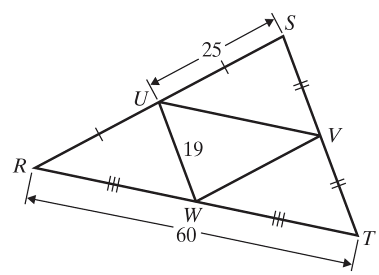
1. Use properties of midsegments of triangle.
2. Use coordinate geometry with midsegments.
3. Solve applications of midsegments.
4. Key vocabulary: *midsegment of a triangle*

Key Examples:

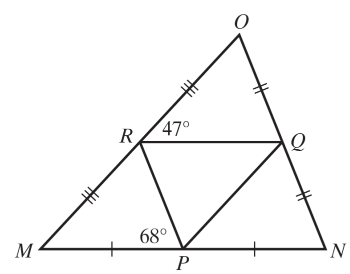
1. a) Name the midsegments and sides that are parallel in ΔABC. Write your answer in the form of three parallel statements.
   b) Name the midsegments that are half as long as the sides of ΔABC. Write your answer in the form of three equations.



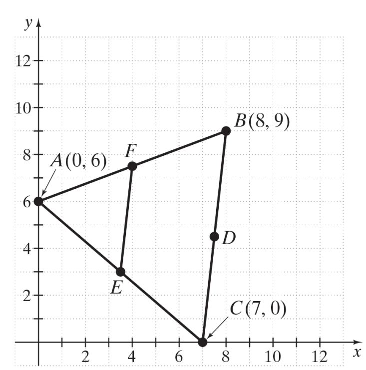
2. In triangle ΔRST, what are the lengths of $\overline{SV}$, $\overline{ST}$, $\overline{VW}$, and $\overline{UV}$?



3. Observe the three pairs of midsegments and sides that are parallel in ΔMNO.
   a) Find $m\angle NMO$.
   b) Find $m\angle PRQ$.



4. Use the given triangle to verify the Triangle Midsegment Theorem.
   a) Find the coordinates of E and F.
   b) Show that $\overline{EF} \parallel \overline{CB}$.
   c) Show that $EF = \dfrac{1}{2}CB$.



Answers: 1a) $\overline{YZ} \parallel \overline{AB}$; $\overline{XZ} \parallel \overline{BC}$; $\overline{XY} \parallel \overline{AC}$ 1b) $YZ = \frac{1}{2}(AB)$, $XZ = \frac{1}{2}(BC)$, $XY = \frac{1}{2}(AC)$ 2) $SV = 19$; $ST = 38$; $VW = 25$; $UV = 30$ 3a) 47° 3b) 68° 4a) $E(3.5, 3)$; $F(4, 7.5)$ 4b) slope of $\overline{EF} = 9$; slope of $\overline{CB} = 9$ 4c) $CB = \sqrt{82} = 2\sqrt{20.5}$; $EF = \sqrt{20.5} = \frac{1}{2}CB$ 5) 185 m

Mini-Lesson 5.4
Midsegments of Triangles

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Interactive Lecture Video Section 5.4

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

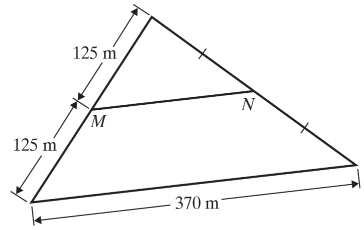
Interactive Lecture Video Objective 3

Video Organizer Section 5.4 (print)

Video Organizer Section 5.4 (MML)

PowerPoints, Section 5.4

5. $\overline{MN}$ represents a sidewalk planned for the park in the figure below. What is the length of the sidewalk?



Teaching Notes:

- Because students may have trouble remembering all the information about the four centers of a triangle from Lessons 5.2 and 5.3, use the Helpful Hint on p.227 of the textbook to review all of this material in one place before introducing the concept of *midsegment.*
- Give students an acute triangle, a right triangle, and an obtuse triangle. Using a ruler, ask them to find the midpoints of the three sides of each triangle and then draw the midsegments, or ask them to construct the three midsegments using compass and straight edge.

ERROR PREVENTION

- Some students are likely to confuse the three *midsegments* of a triangle with the three *medians.* To help them avoid this error, compare the definitions of *midsegment* and *median.* Then ask them why a midsegment can never be a median.

Closure Questions:

- In what type of triangle are exactly two of the midsegments congruent? In what type of triangle are all three midsegments congruent?

  Exactly two of the midsegments are congruent in an isosceles triangle that is not equilateral. All three midsegments are congruent in an equilateral triangle.

- What is the relationship between the perimeter of the triangle formed by the three midsegments of a given triangle and the perimeter of the original triangle? Explain.

  The perimeter of the triangle whose sides are the three midsegments of a given triangle is half the perimeter of the original triangle because each midsegment is half the length of the side to which it is parallel.

Mini-Lesson 5.5

Indirect Proofs and Inequalities in One Triangle

Learning Objectives:

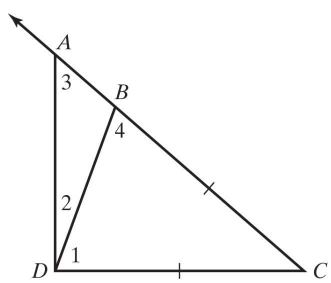
1. Use indirect reasoning to write proofs.
2. Learn the triangle relationship between length of a side and size of its opposite angle.
3. Use the Triangle Inequality Theorem.
4. Key vocabulary: *indirect reasoning, indirect proof*

Key Examples:

1. Suppose you want to write an indirect proof of each statement. As the second step of the proof, what would you assume in each case?
   a) The next nearest town is at least 15 miles away.
   b) ΔABC is equilangular.

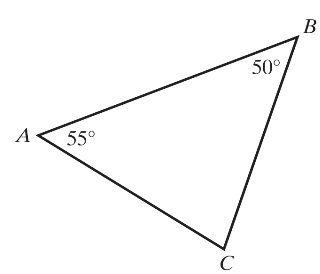
2. Which two statements contradict each other?
   I. In ΔABC , $m\angle A < m\angle B + m\angle C$
   II. In ΔABC , $m\angle B = 90°$
   III. ΔABC is scalene.

3. In the figure, why is $m\angle 4 > m\angle 1$?



4. A triangular lot has sides equal to 120 meters, 150 meters, and 200 meters. Which two sides form the corner with the smallest angle?

5. In the figure at the right, $m\angle A = 55°$ and $m\angle B = 50°$.
   List the sides of ΔABC from shortest to longest.



Answers: 1a) Assume temporarily that the next town is less than 15 miles away. 1b) Assume temporarily that ΔABC is not equiangular. 2) II and III 3) $\angle 4$ is an exterior angle of ΔABD, so by the Corollary to the Triangle Exterior Theorem, $m\angle 4 > m\angle 3$. $\overline{BC} \cong \overline{DC}$, so $\angle 1 \cong \angle 4$ by the Isosceles Base Angles Theorem, and $m\angle 1 = m\angle 4$. Therefore, $m\angle 1 > m\angle 3$ by substitution. 4) the 150-m and 200-m sides 5) $\overline{AC}$, $\overline{BC}$, $\overline{AB}$ 6a) No; 13 + 15 = 28 < 29 6b) Yes; the sum of the lengths of any two sides is greater than the length of the third side. 7) Greater than 12 cm and less than 42 cm

Mini-Lesson 5.5

Indirect Proofs and Inequalities in One Triangle

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eText, Section 5.5 (MML)

Interactive Lecture Video Section 5.5

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Interactive Lecture Video Objective 3

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Video Organizer Section 5.5 (MML)

PowerPoints, Section 5.5

6. Can a triangle have sides with the given lengths? Explain.
   a) 13 m, 15 m, 29 m
   b) 24 in., 36 in., 54 in.

7. A triangle has side lengths of 15 cm and 27 cm. What is the range of possible lengths for the third side?

Teaching Notes:

- The concept and process of indirect proof can be very confusing to students. Start gradually and include several examples from everyday (non-mathematical) contexts. Have students practice the assumption step (Step 2) with specific examples before moving on to Steps 3 and 4.

ERROR PREVENTION

- If students are having trouble writing indirect proofs, pinpoint which step of the process is giving them trouble and then provide extra practice with that step. Require that students number the steps of their indirect proofs to make it easier to pinpoint errors or sources of confusion.

Closure Questions:

- Why is indirect proof also called "proof by contradiction"?

  In an indirect proof, we assume that the opposite of the statement to be proved is true and show that this leads to a contradiction.

- How is the statement "the shortest distance between two points in on a straight line" related to the Triangle Inequality Theorem?

  In any triangle ABC, the shortest distance from A to C is AC (the length of $\overline{AC}$), while AB + BC is the length of a longer path between the same two points. This statement is equivalent to the Triangle Inequality Theorem: AB + BC > AC.

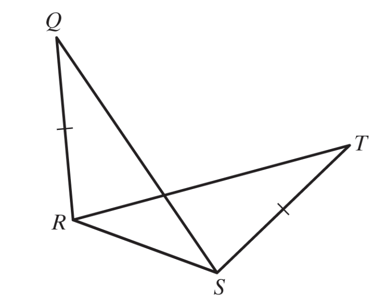
Mini-Lesson 5.6
Inequalities in Two Triangles

Learning Objectives:

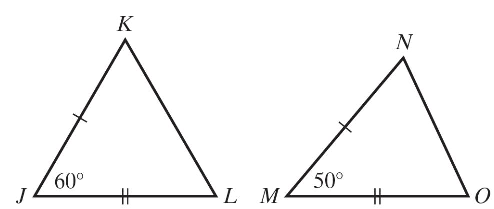
1. Use the Hinge Theorem and its converse to compare measures of sides and angles of two triangles.

Key Examples:

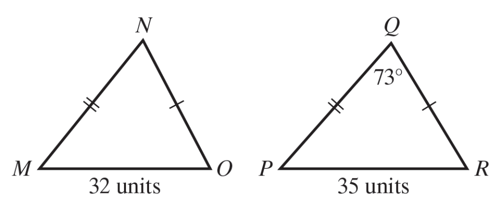
1. **Given:** $QR = ST$, $m\angle TSR > m\angle QRS$
   Prove: $RT > QS$



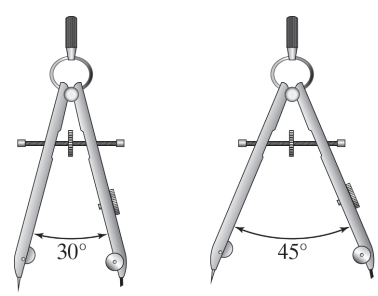
2. **Multiple Choice** Which of the following statements must be true?
   A. $JK < NO$
   B. $KL > NO$
   C. $KL < NO$
   D. $JL = NO$



3. Choose the only possible measure for $\angle N$.
   a) 65°
   b) 75°
   c) 85°



4. The diagram at the right shows a compass in two different positions. In which position is the distance between the tips of the two points greater? Use the Hinge Theorem to justify your answer.



Answers: 1) *See Additional Answers at end of Mini-Lessons*. 2) B 3) a 4) The 45° opening; the lengths of the compass legs do not change as the compass is opened. The included angle between the points at the 45° opening is greater than the included angle of the 30° opening, so by the Hinge Theorem, the distance between the tips is greater for the 45° opening. 5) $25 < x < 97.5$

Mini-Lesson 5.6

Inequalities in Two Triangles

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Interactive Lecture Video Section 5.6

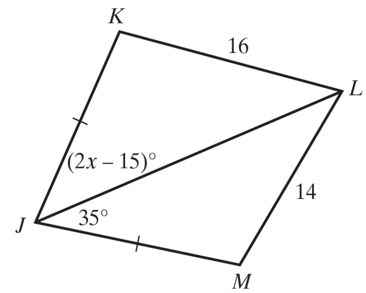
Interactive Lecture Video Objective 1

Video Organizer Section 5.6 (print)

Video Organizer Section 5.6 (MML)

PowerPoints, Section 5.6

5. What is the range of possible values for x?



Teaching Notes:

- Introduce the Hinge Theorem with physical objects. The drawings of a door hinge on p.242 of the textbook, scissors on p.244, and a robotic arm and lamp in Exercises 17 and 18 on p.246 are all excellent examples. To enhance the power of these drawings, illustrate the theorem with actual objects, such as using the hinge on the door of your classroom or bringing in a pair of scissors or other suitable objects.
- Ask students why the Hinge Theorem is also called the SAS Inequality Theorem, while its converse is also called the SSS Inequality Theorem. Discuss how these theorems are similar to and different from the SAS and SSS Congruence Theorems.

ERROR PREVENTION

- In Example 5, some students may think that the answer should be "$x = 16$ or $x = 4$" rather than the compound inequality $4 < x < 16$. This is understandable because when they have worked with figures marked like this one in earlier chapters, the answer was just one specific value of x. Although the figure for this example doesn't suggest a range of values, we need to know the lower and upper limit for a range of values.

Closure Questions:

- Can the Hinge Theorem be applied to a pair of equilateral triangles? Explain.

  No; In every equilateral triangle, all three angles measure 60°, so the part of the hypothesis of the Hinge Theorem that says "and the included angles are not congruent" does not apply.