Mini-Lesson 8.1
Rigid Transformations

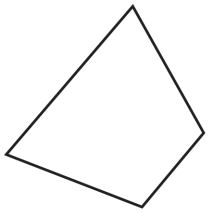
Learning Objectives:

1. Identify rigid transformations or isometries.
2. Name images and corresponding parts.
3. Key vocabulary: *transformation, preimage, image, isometry, rigid transformation*

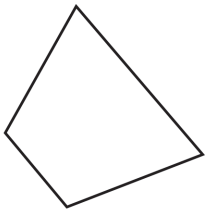
Key Examples:

1. Does each transformation appear to be an isometry? Explain.

a)

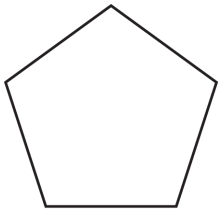


Preimage

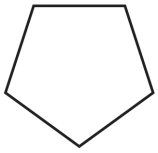


Image

b)



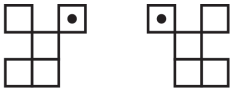
Preimage



Image

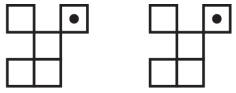
2. Identify the single transformation from the preimage to each individual image.

a)



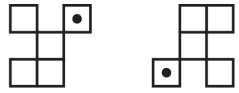
Preimage Image

b)



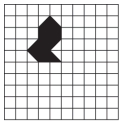
Preimage Image

c)



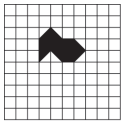
Preimage Image

3. Identify the single transformation from the preimage to each individual image.



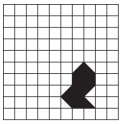
Preimage

a)



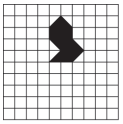
Image

b)



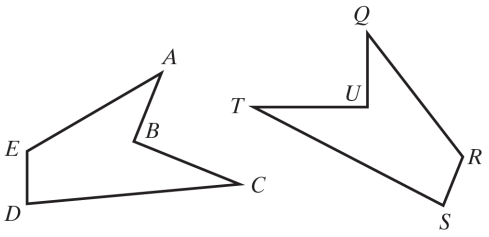
Image

c)



Image

4. In the diagram, $ABCDE \rightarrow QUTSR$.
   a) What are the images of point B and of point C?
   b) What are the pairs of congruent corresponding sides?



Answers: 1a) Yes, the preimage and image appear to be congruent. 1b) No, the figures are not the same size. 2a) reflection 2b) translation 2c) rotation 3a) rotation 3b) translation 3c) reflection 4a) point U; point T 4b) $\overline{AB} \cong \overline{QU}$; $\overline{BC} \cong \overline{UT}$; $\overline{CD} \cong \overline{TS}$; $\overline{DE} \cong \overline{SR}$; $\overline{EA} \cong \overline{RQ}$

Mini-Lesson 8.1

Rigid Transformations

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 8.1 (MML)

Interactive Lecture Video Section 8.1

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 8.1 (print)

Video Organizer Section 8.1 (MML)

PowerPoints, Section 8.1

Teaching Notes:

- Emphasize "mapping" vocabulary with arrow notation. Stress that under an isometry, the preimage and image are congruent figures, so corresponding vertices should be written in corresponding positions in an isometry statement, just as in a congruence statement.
- The word *isometry* may be unfamiliar to students. To help them remember it and understand its meaning, tell them that the prefix *iso-* means "equal," while the suffix *-metry* means "measure." Then ask them what other words they can think of that have the same prefix or suffix. For *iso-,* they are likely to think of *isosceles* ("equal legs"), and possibly terms from science courses such as *isotope, isomer,* and *isobar.* Ask if they are familiar with *isometric exercises.* For *-metry*, they should come up with *geometry* ("earth measure"), *symmetry,* and possibly *trigonometry* ("triangle measure"), which they will study in Chapter 9.

ERROR PREVENTION

- If students have trouble distinguishing between translations, reflections, and rotations, demonstrate dynamically with physical objects or a computer animation the actions necessary to go from the preimage to the image.

Closure Questions:

- What are the mathematical terms for slides, flips, and turns?

  translations, reflections, and rotations

- How are geometric reflections related to reflection in a mirror?

  The image of an object under a geometric reflection is its "mirror image" on the other side of a line.

.

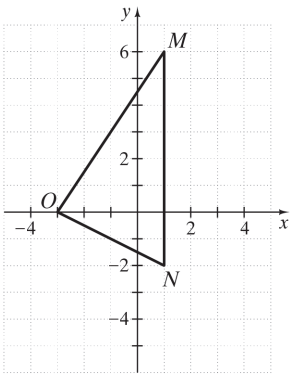
Mini-Lesson 8.2
Translations

Learning Objectives:

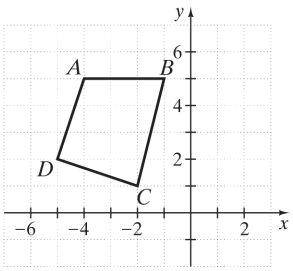
1. Find translation images of figures.
2. Key vocabulary: *translation, composition of transformations*

Key Examples:

1. a) Find the image of each vertex of ΔMNO for the translation $(x, y) \rightarrow (x + 3, y - 3)$.
   b) Graph ΔMNO and its image.



2. The translation image of $ABCD$ is $A'B'C'D'$ with $A'(2, 3)$, $B'(5, 3)$, $C'(4, -1)$, and $D'(1, 0)$. What is a translation rule that describes the translation?



3. On Tuesdays and Thursdays, a college student has a math class, followed by a history class, and lastly a chemistry class. The shortest route between the buildings in which the mathematics the history departments are located is 1 block south and 4 blocks west. The chemistry building is 3 blocks east and 1 block south of the history building. Where is the chemistry building in relation to the mathematics building?

Answers: 1a) $M'(4, 3)$, $N'(4, -5)$, $O'(0, -3)$ 1b) *See Additional Answers at end of Mini-Lessons*.
2) $(x, y) \rightarrow (x + 6, y - 2)$ 3) 1 block west and 2 blocks south

Mini-Lesson 8.2

Translations

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 8.2 (MML)

Interactive Lecture Video Section 8.2

Interactive Lecture Video Objective 1

Video Organizer Section 8.2 (print)

Video Organizer Section 8.2 (MML)

PowerPoints, Section 8.2

Teaching Notes:

- Require students to draw translations accurately using either graph paper or an electronic grid for online work. The experience of actually drawing the figures themselves (rather than only using online tools) is helpful in building understanding. This applies to the work with coordinates throughout this chapter.
- Provide practice in going back and forth between writing a translation rule symbolically and stating in words what the translation does to each point on the preimage.

ERROR PREVENTION

- If students have trouble with recognizing or performing translations, guide them in breaking down the process by asking themselves these questions: Direction: right or left? up or down? Distance: How many units in each direction?

Closure Questions:

- What is the result of the composition of the translations $(x, y) \rightarrow (x + 4, y - 3)$ and $(x, y) \rightarrow (x - 4, y + 3)$? How would you describe the relationship between these two translations?

  The composition is $(x, y) \rightarrow (x, y)$. The result of the two translations one after the other is the original preimage. I would call these "opposite translations."

Mini-Lesson 8.3
Reflections

Learning Objectives:

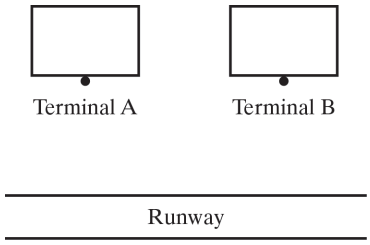
1. Find reflection images of figures.
2. Identify line symmetry.
3. Key vocabulary: *reflection, line of reflection, line symmetry, reflectional symmetry line of symmetry*

Key Examples:

1. What is the image of P (7, –5) reflected across the line $y = 2$?

2. Graph ΔJKL, where J (3, 4), K (–3, 1), and L (–1, –3). What is the image of ΔJKL reflected across the x-axis?

3. Two taxi strips are to be constructed between two terminals, A and B, and a main runway. At what point C along the runway should the strips be built so that the total length of the strips is minimized?



4. How many lines of symmetry does a regular pentagon have?

Answers: 1) (7, 9) 2) and 3) *See Additional Answers at end of Mini-Lessons.* 4) five

Mini-Lesson 8.3

Reflections

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 8.3 (MML)

Interactive Lecture Video Section 8.3

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 8.3 (print)

Video Organizer Section 8.3 (MML)

PowerPoints, Section 8.3

Teaching Notes:

- The MIRA™ geometry tool is very helpful in studying reflections, as well as other types of transformations. This could be used for demonstrations, but is particularly effective if students have access to working with MIRAs individually or in groups.
- Remind students that the distance from a point to a line is defined as the perpendicular distance.
- Emphasize we are reflecting a figure over a *line,* so it is critical to identify the line of reflection. Although all the examples and exercise with coordinates in this lesson have one of the axes or another horizontal or vertical line as the line of reflection, it is possible to reflect a figure over any line in the plane.

ERROR PREVENTION

- Some students may make errors by confusing the equations of horizontal and vertical lines. Provide examples to help them review the concepts that every vertical line has an equation of the form $x = a$ because every point on the line has the same x-coordinate, while every horizontal line has an equation of the form $y = b$ because every point on the line has the same y-coordinate.

Closure Questions:

- If a point on the preimage of a figure lies on the line of reflection, what do you know about its reflection image?

  The image will be the same point as the preimage.

- If you draw a figure on paper and cut it out, how can test it for lines of symmetry?

  Fold the figure on a line. If the pieces of the figure on the two sides of the line coincide, the fold line is a line of symmetry.

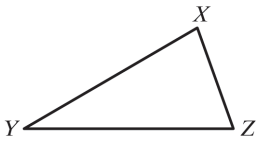
Mini-Lesson 8.4
Rotations

Learning Objectives:

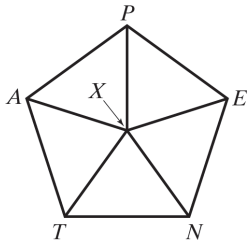
1. Draw and identify rotation images of figures.
2. Identify rotational symmetry.
3. Key vocabulary: *rotation, center of rotation, angle of rotation, center of a regular polygon, symmetry, rotational symmetry*

Key Examples:

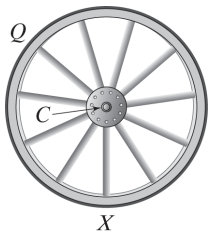
1. What is the image of ΔXYZ for a 45° rotation about Z ?



2. What is the image of P for a 288° rotation about X ?



3. For the wheel in the figure, what is the angle of rotation about C that maps Q to X?



4. What are the coordinates of the image of point X (7, 7) for a composition of a 60° rotation and a 75° rotation about the origin?

Answers: 1) *See Additional Answers at end of Mini-Lessons.* 2) *E* 3) 130.9° 4) (–7, 0) 5) No

Mini-Lesson 8.4

Rotations

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 8.4 (MML)

Interactive Lecture Video Section 8.4

Interactive Lecture Video Objective 1

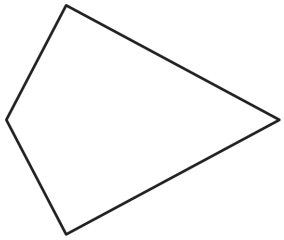
Interactive Lecture Video Objective 2

Video Organizer Section 8.4 (print)

Video Organizer Section 8.4 (MML)

PowerPoints, Section 8.4

5. Does the figure have rotational symmetry? If so, what is the angle of rotation?



Teaching Notes:

- Many students find rotations more difficult to visualize and work with than translations and reflections, particularly when drawing a rotation image, whether or not they are working with coordinates. Discuss Example 1 on p.353 of the textbook carefully and provide extra practice in using this procedure for students who need it.

ERROR PREVENTION

- Students may have trouble visualizing rotations shown on paper. A dynamic demonstration with actual objects or a computer animation will be helpful, as this will allow students to actually see the motion.

Closure Questions:

- If you draw a figure on paper and cut it out, how can test it for rotational symmetry?

  Make a copy of the figure on a second sheet of paper. Cut out one of the figures and place it on top of the other one. Rotate the figure you cut own and see which angles of rotation will result in the two figures coinciding.

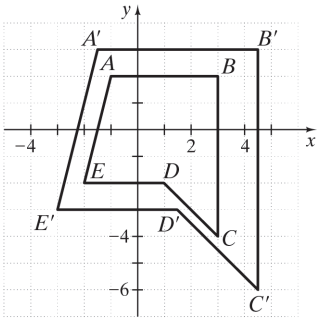
Mini-Lesson 8.5
Dilations

Learning Objectives:

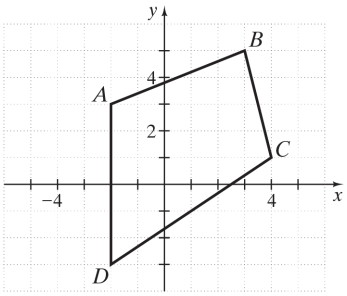
1. Understand dilation images of figures.
2. Key Vocabulary: *dilation, center of dilation, scale factor of a dilation, enlargement, reduction*

Key Examples:

1. *A'B'C'D'E'* is a dilation image of *ABCDE*. The center of dilation is the origin *O*. Is the dilation an enlargement or a reduction? What is the scale factor of the dilation?



2. What are the images of the vertices of *ABCD* for a dilation with center (0, 0) and scale factor $\frac{1}{3}$?



3. A paramecium (a type of one-celled organism) is 0.25 mm long. What is the length of this organism when viewed under a microscope that magnifies objects by a scale factor of 200?

Answers: 1) enlargement; 1.5 2) $A'\left(-\frac{2}{3},\ 1\right)$, $B'\left(1,\ \frac{5}{3}\right)$, $C'\left(\frac{4}{3},\ \frac{1}{3}\right)$, $D'\left(-\frac{2}{3},\ -1\right)$ 3) 50 mm or 5 cm

Mini-Lesson 8.5

Dilations

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 8.5 (MML)

Interactive Lecture Video Section 8.5

Interactive Lecture Video Objective 1

Video Organizer Section 8.5 (print)

Video Organizer Section 8.5 (MML)

PowerPoints, Section 8.5

Teaching Notes:

- Start this section by asking students to think of examples in everyday life that involve enlarging or reducing a two-dimensional or three-dimensional object without changing its shape.
- Do an in-class computer demonstration of enlarging and reducing a figure using different scale factors or find a computer program that allows students to do this themselves. Use computer art to show what happens if you enlarge or reduce a figure using different scale factors for the vertical and horizontal distances. Make sure that students understand that this is *not* a dilation.
- Notice that all examples and exercises in this section of the textbook involving dilations with coordinates use the origin as the center of dilation. If a different center were used, the work would be more difficult.

ERROR PREVENTION

- Some students may not understand the importance of the *center of dilation.* The figure in the "Dilation" box on p.360 of the textbook is very helpful in explaining this concept. Redraw the figure using a different point than C as the center of dilation, ask students to find the image of ΔPQR with the new center and to compare it to the image triangle shown in the textbook.

Closure Questions:

- If a figure is dilated with the origin as the center of dilation, what will be the image of a point (x, y) on the figure if the scale factor is n?

  (nx, ny)

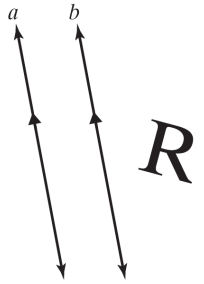
Mini-Lesson 8.6
Compositions of Reflections

Learning Objectives:

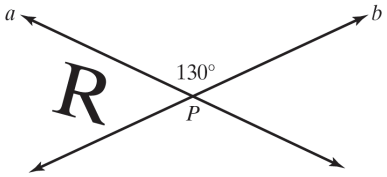
1. Find compositions of reflections, including glide reflections.
2. Classify isometries.
3. Key vocabulary: *glide reflection*

Key Examples:

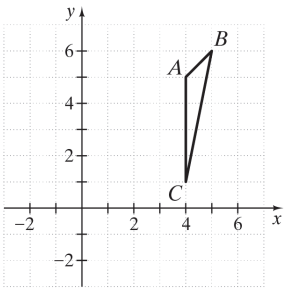
1. What is the image of R reflected first across line *a* and then across line *b*?
   What are the direction and distance of the resulting translation?



2. What is the image of R reflected first across line *b* and then across line *a*?
   What are the center of rotation and the angle of rotation for the resulting rotation?

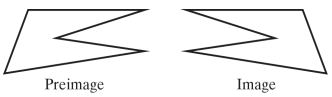


3. What is the image of ΔABC for a glide reflection where the translation is $(x, y) \rightarrow (x - 7, y)$ and the line of reflection is $y = 1$?



4. Each transformation is an isometry. Are the orientations of the preimage and image the same or opposite? What type of isometry maps the preimage to the image?

a)



b)

Answers: 1)–3) *See Additional Answers at end of Mini-Lessons*. 4a) opposite; reflection 4b) opposite; glide reflection

Mini-Lesson 8.6

Compositions of Reflections

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 8.6 (MML)

Interactive Lecture Video Section 8.6

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 8.6 (print)

Video Organizer Section 8.6 (MML)

PowerPoints, Section 8.6

Teaching Notes:

- The material in this section can be quite difficult for students, so allow plenty of time to go through it step-by-step. Explain that we are bringing together translations, rotations, and reflections, but are not working with dilations because they are not isometries.
- Emphasize the concept of *orientation* throughout this section. When composing reflections, ask students at each stage whether the orientation of the figure is the same as before or has changed to the opposite orientation.

ERROR PREVENTION

- Students are likely to have trouble with glide reflections. The illustration with the paw prints on p.267 of the textbook is very helpful in explaining this concept. When students work with glide reflections, emphasize that this is a two-step process: first perform the translation, and then perform the reflection on the image from the translation, *not* on the original figure.

Closure Questions:

- Which types of isometries preserve the orientation of a figure? Which reverse the orientation?

  Translations and rotations preserve the orientation of a figure. Reflections and glide reflections reverse it to give the opposite orientation from the original figure.