Mini-Lesson 10.1

Angle Measures of Polygons and Regular Polygon Tessellations

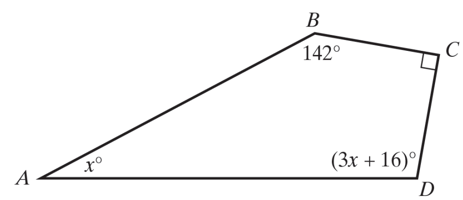
Learning Objectives:

1. Find the measures of interior angles of polygons.
2. Find the measures of exterior angles of polygons.
3. Determine whether a tessellation of regular polygons is formed.
4. Key vocabulary: *exterior angles of the polygon, tessellation*

Key Examples:

1. Find the sum of the measures of the interior angles of a convex heptagon.

2. Find the value of x in the figure.
   Then use x to find $m\angle A$ and $m\angle D$.

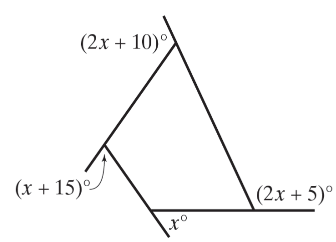


3. Find the measure of each interior angle of a regular dodecagon.

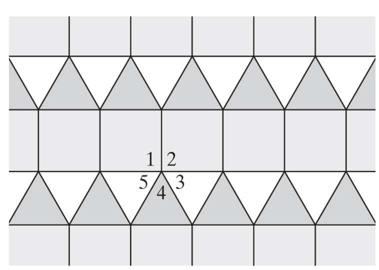
4. The measure of an interior angle of a regular polygon is 165°. Find the number of sides of this polygon.

5. Find the measure of each exterior angle in a regular 30-gon.

6. Find the value of x.
   Then find each exterior angle measure.



7. Check to see whether this is a tessellation by finding the sum of the numbered angles of the regular polygons.



Answers: 1) 900° 2) $x = 28$, $m\angle A = 28°$, $m\angle D = 100°$ 3) 150° 4) 24 sides 5) 12° 6) $x = 55$; $x° = 55°$, $(x + 15)° = 70°$, $(2x + 5)° = 115°$, $(2x + 10)° = 120°$ 7) $m\angle 1 = m\angle 2 = 90°$, $m\angle 3 = m\angle 4 = m\angle 5 = 60°$; $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360°$; yes, this is a tessellation.

Mini-Lesson 10.1

Angle Measures of Polygons and Regular Polygon Tessellations

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 10.1 (MML)

Interactive Lecture Video
Section 10.1

Interactive Lecture Video
Objective 1

Interactive Lecture Video
Objective 2

Interactive Lecture Video
Objective 3

Video Organizer
Section 10.1 (print)

Video Organizer
Section 10.1 (MML)

PowerPoints, Section 10.1

Teaching Notes:

- Ask students to come up with their own ways to remember that the sum of the measures of the interior angles of an n-gon is $(n - 2) \bullet 180°$.

ERROR PREVENTION

- Some students may try to apply the formulas for the measure of an interior or exterior angle of a polygon to a polygon that is not regular. Ask these students to define *regular polygon.* This should help them to see that these formulas can't work unless all interior angles of the polygon are congruent.

Closure Questions:

- Is there any type of polygon for which all the interior angles and all the exterior angles are congruent to each other? If so, must it be a regular polygon? Explain.

  This is true for any rectangle because all of the interior angles and all of the exterior angles are right angles. If this were true for a regular polygon, it would be a square, but it is true for all rectangles, whether they are squares of not.

- In a tessellation made up of polygons, why must the sum of the measures of the angles at every vertex be 360°?

  If the sum of the angle measures is less than 360°, there will be a gap at the vertex between the polygons. If the sum of the angle measures is greater than 360°, the polygons will overlap at the vertex. Neither of these situations is allowed in a tessellation.

Mini-Lesson 10.2

Areas of Triangles and Quadrilaterals with a Review of Perimeter

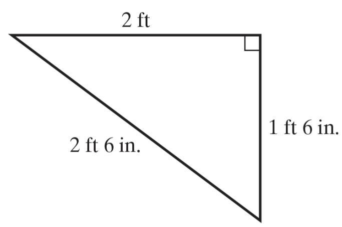
Learning Objectives:

1. Find areas of squares, rectangles, parallelograms, and triangles.
2. Find areas of trapezoids, rhombuses, and kites.
3. Key vocabulary: *base of a parallelogram, height of a parallelogram, base of a triangle, height of a triangle, height of a trapezoid*

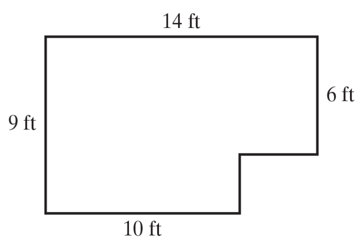
Key Examples:

1. What is the area of a parallelogram with base 17 cm and height 5.5 cm?

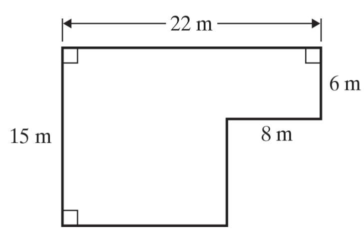
2. What is the area of the triangle in square inches?



3. Find the perimeter of the room shown in the figure.

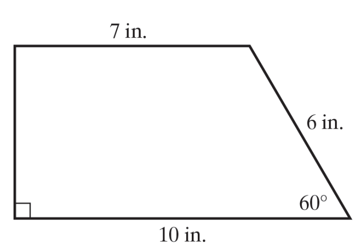


4. Find the area of the figure.



5. What is the area of a trapezoid with bases 18 yards and 11 yards and height 9 yards?

6. Find the area of the trapezoid shown in the figure.



Answers: 1) 93.5 sq cm 2) 216 sq in. 3) 46 ft 4) 258 sq m 5) 130.5 sq yd 6) $25.5\sqrt{3}$ sq in. 7) 195 sq cm

Mini-Lesson 10.2

Areas of Triangles and Quadrilaterals with a Review of Perimeter

1

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 10.2 (MML)

Interactive Lecture Video Section 10.2

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 10.2 (print)

Video Organizer Section 10.2 (MML)

PowerPoints, Section 10.2

7. Find the area of a kite with diagonals that are 15 cm and 26 cm long.

Teaching Notes:

- The justifications of the formulas for the area of a parallelogram and a triangle on p.437 of the textbook are more effective if students are able to view them dynamically. Use demonstrations that show the movement of a triangle from a parallelogram to form a rectangle and splitting a parallelogram into two triangles (or copying a triangle to form a parallelogram).

ERROR PREVENTION

- In a problem like Example 2 on p.437 of the textbook, students may make errors by failing to convert measurements to a common unit. This could happen either by not noticing that the units differ or by not converting correctly. Remind students that whenever they are asked to find the perimeter or area of a figure in which the measurements are given with units, they must first check to see whether all of the units match and then, if necessary, to convert to a common unit.

Closure Questions:

- Can the formulas for the area of a rectangle, a rhombus, and a parallelogram be used to find the area of a square? Explain.

  Yes, but the formula for the area of a square is simpler than any of the other area formulas. The formula for the area of a square is just the special case of the formula for the area of a rectangle or parallelogram with $s = b = h$. Since a square is also a rhombus, the rhombus formula could be used if the length of the diagonal is given rather than the side length.

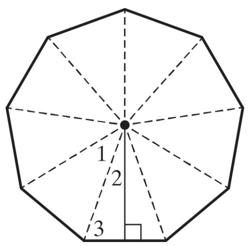
Mini-Lesson 10.3
Areas of Regular Polygons

Learning Objectives:

1. Find the area of a regular polygon.
2. Find the area of a regular polygon using trigonometric ratios.
3. Key vocabulary: *center of a regular polygon, radius of a regular polygon, apothem, central angle of a regular polygon*

Key Examples:

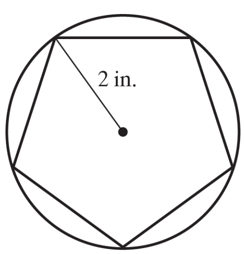
1. The figure is a regular nonagon with radii and an apothem drawn. What is the measure of each numbered angle?



2. What is the area of a regular octagon with a 12.1-inch apothem and 10-inch sides?

3. The side of an equilateral triangle is 8 cm. What is the area of the triangle? Round the answer to the nearest square centimeter.

4. The design of a scouting badge uses a circle with radius 2 inches circumscribed about a regular pentagon. Find the area of the pentagon rounded to the nearest tenth of a square inch.



5. What is the area of a regular decagon with 24-mm sides? Round the answer to the nearest square millimeter.

Answers: 1) $m\angle 1 = 40°$, $m\angle 2 = 20°$, $m\angle 3 = 70°$ 2) 484 sq in. 3) 28 sq cm 4) 9.5 sq in. 5) 4432 square mm

Mini-Lesson 10.3

Areas of Regular Polygons

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 10.3 (MML)

Interactive Lecture Video Section 10.3

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 10.3 (print)

Video Organizer Section 10.3 (MML)

PowerPoints, Section 10.3

Teaching Notes:

- Emphasize that all of the work in this section involves regular polygons only. Make sure that students understand all of the vocabulary illustrated at the bottom of p.445 of the textbook and defined on the top of p.446 before discussing any of the examples.
- The word *apothem* is probably unfamiliar to students. In addition to defining this term and illustrating it with a few different regular polygons, tell students that the accent is on the first syllable. Also tell them that, as with *radius* and *diameter,* the term *apothem* can be used to refer to either the segment or its length.

ERROR PREVENTION

- Some students may leave out the factor of $\frac{1}{2}$ in the formula $A=\frac{1}{2}ap$ for the area of a regular polygon. Remind them that the radii of an n-gon divide the polygon into n isosceles triangles. The areas of these triangles are added to get the area of the polygon, so the factor of $\frac{1}{2}$ in the triangle area formula also appears in the formula for the area of a regular polygon.

Closure Questions:

- How can you find the measure of a central angle of a regular polygon? Describe how you would do this in words and then write an expression for the measure of a central angle of an n-gon.

  Divide 360° by the number of sides. The measure of a central angle of an n-gon is $\frac{360^\circ}{n}$.

- For which regular polygons can you use special triangles to find the apothem?

  equilateral triangle, square, and regular hexagon

Mini-Lesson 10.4

Perimeters and Areas of Similar Figures

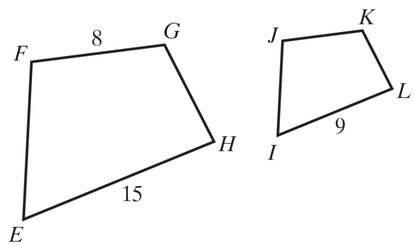
Learning Objectives:

1. Find the perimeters and areas of similar figures.
2. Key vocabulary: *scale factor*

Key Examples:

1. Figure *EFGH* ~ figure *IJKL*.

   a) Find the scale factor of the larger figure to the smaller figure.
   b) Given the scale factor, find *JK*.
   c) Find the ratio of the perimeter of the larger figure to the smaller figure.
   d) Find the ratio of the area of the smaller figure to the larger figure.



2. The scale factor of two similar trapezoids is $\frac{7}{4}$. The area of the larger trapezoid is 98 square centimeters. What is the area of the smaller trapezoid?

3. The scale factor of the dimensions of two similar offices is 2:3. The cost for new carpeting for the smaller office is \$250. How much would the same type of carpeting cost for the larger office?

4. The areas of two similar hexagons are 486 ft^2 and 150 ft^2. What is the ratio of their perimeters?

Answers: 1a) $\frac{5}{3}$ 1b) 4.8 1c) $\frac{5}{3}$ or 5:3 1d) $\frac{9}{25}$ or 9:25 2) 32 sq cm 3) \$562.50 4) $\frac{9}{5}$ or 9:5

Mini-Lesson 10.4

Perimeters and Areas of Similar Figures

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 10.4 (MML)

Interactive Lecture Video Section 10.4

Interactive Lecture Video Objective 1

Video Organizer Section 10.4 (print)

Video Organizer Section 10.4 (MML)

PowerPoints, Section 10.4

Teaching Notes:

- Begin this section with a brief review of similar polygons from Chapter 7, emphasizing that the corresponding parts of similar polygons are *proportional.* Rather than telling students the conclusions of Theorem 10.4-1 on p.452 of the textbook, give them the opportunity to discover these relationships by creating their own examples.

ERROR PREVENTION

- A common error when working with this topic is for students to forget to square the scale factor when calculating the area of one figure given the area of a similar figure. Students will remember this concept better if they see it illustrated with geometric figures. For example, start with a rectangle with length 2 cm and width 3 cm. Then double both the length and the width to form a larger rectangle. Ask students to calculate the area of each rectangle and compare the results.

Closure Questions:

- When comparing measurements of similar figures, why do we square the scale factor to find the ratio of their areas, but not to find the ratio of their perimeters?

  Perimeter is a one-dimensional measurement. The perimeter of a figure is the sum *of the lengths of the sides, so if you double each length of each side, the perimeter will double. Area is a two-dimensional measurement that involves a* product *of side lengths, so if you double each length, the area will be multiplied by 4.*

Mini-Lesson 10.5

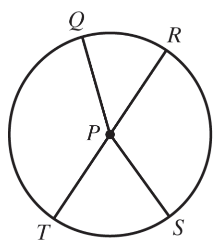
Arc Measures, Circumferences, and Arc Lengths of Circles

Learning Objectives:

1. Find the measures of central angles and arcs.
2. Find the circumference and arc length.
3. Key Vocabulary: *circle, center, diameter, radius, congruent circles, central angle, semicircle, minor arc, major arc, adjacent arcs, circumference, pi, concentric circles, arc length, congruent arcs*

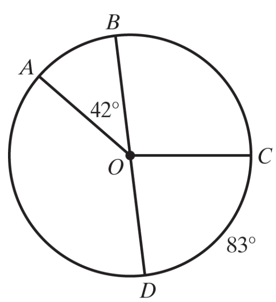
Key Examples:

1. a) What are the minor arcs of $\odot P$?
   b) What are the semicircles of $\odot P$?
   c) What are the major arcs of $\odot P$ that contain point *S*?



2. What are the measures of each arc in $\odot O$?

   a) $\overset{\frown}{AB}$ b) $\overset{\frown}{BC}$
   c) $\overset{\frown}{ABD}$ d) $\overset{\frown}{CDB}$



3. A 4-inch-diameter drill blade has a $3\frac{1}{8}$-inch-diameter hole. Approximate the difference in the circumferences to the nearest hundredth of an inch.

4. What is the length of a semicircle with diameter 8.6 m? Leave your answer in terms of π.

Answers: 1a) $\overset{\frown}{QR}$, $\overset{\frown}{QS}$, $\overset{\frown}{RS}$, $\overset{\frown}{ST}$, $\overset{\frown}{TQ}$ 1b) $\overset{\frown}{TQR}$, $\overset{\frown}{RST}$ 1c) $\overset{\frown}{TQS}$, $\overset{\frown}{QRT}$, $\overset{\frown}{RSQ}$, $\overset{\frown}{STQ}$, $\overset{\frown}{STR}$ 2a) 42° 2b) 97° 2c) 222° 2d) 263° 3) 2.75 in. 4) 4.3π m

Mini-Lesson 10.5

Arc Measures, Circumferences, and Arc Lengths of Circles

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 10.5 (MML)

Interactive Lecture Video Section 10.5

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 10.5 (print)

Video Organizer Section 10.5 (MML)

PowerPoints, Section 10.5

Teaching Notes:

- This section has a very long vocabulary list. Start this section by presenting the entire list on p.457 in the textbook and asking students to define the terms they already know and/or illustrate them with diagrams. Then, the list of new terms to learn will seem less overwhelming.
- Give your students clear instructions on whether they can use approximations such as 3.14 and $\frac{22}{7}$ in calculations involving π or should always use the π key on their calculators, which gives more accurate results.

ERROR PREVENTION

- When asked what "pi" is, some students will answer "3.14." Explain to them that 3.14 and $\frac{22}{7}$ are both *rational* numbers that common *approximations* for the *irrational* number π..

Closure Questions:

- Why does the number π appear in many formulas and problems involving circles?

  The number π is the ratio of the circumference to the diameter of any circle.

Mini-Lesson 10.6

Areas of Circles and Sectors

Learning Objectives:

1. Find the areas of circle, sectors, and segments of circles.
2. Key vocabulary: *sector of a circle, segment of a circle*

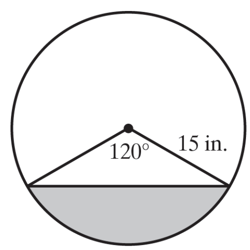
Key Examples:

1. a) A circle has diameter 26 in. Find the exact area, and then a two-decimal-place approximation.
   b) A circle has area 215 sq m. Find the exact radius, and then a two-decimal place approximation.

2. The General Grant tree, a giant sequoia in Kings Canyon National Park in California, is one of the largest trees in the world. Its girth (circumference) at the base is 107.6 feet. Find the area of a slice of this tree near the base to the nearest square foot.

3. A circle has a radius of 24 cm. Find the exact area of a sector bounded by a 30° minor arc. Then give a two-decimal place approximation.

4. Find the area of the shaded segment shown in the figure. Round your answer to the nearest tenth.



Answers: 1a) 169π sq in.; 530.93 sq in. 1b) $\sqrt{\frac{215}{\pi}}$ sq m; 8.27 sq m 2) 921 sq ft 3) 48π sq cm; 150.80 sq cm 4) 138.2 sq in.

Mini-Lesson 10.6

Areas of Circles and Sectors

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 10.6 (MML)

Interactive Lecture Video Section 10.6

Interactive Lecture Video Objective 1

Video Organizer Section 10.6 (print)

Video Organizer Section 10.6 (MML)

PowerPoints, Section 10.6

Teaching Notes:

- Start with the justification of the area formula for a circle shown on p.464 of the textbook. This approach will demonstrate the relationship between the area and circumference of a circle and why the area formula contains π. This geometric justification will be more effective if you can present it dynamically rather than just on paper.

ERROR PREVENTION

- Students are likely to be confused by the difference between a *segment* of a circle and a *sector* of a circle. They will probably remember which is which most easily by using sketches. It may also be helpful to point out that a sector is a "piece" of a circle, bounded by two radii and an arc (shaped like a piece of pie), while a segment is bounded by one side of a triangle (a *segment*) and an arc.

Closure Questions:

- Write a formula for the area of a semicircle. Why isn't it necessary to memorize this formula?

  $A = \frac{1}{2}\pi r^2$; it is not necessary to memorize this formula because the area of a semicircle is half the area of the circle.

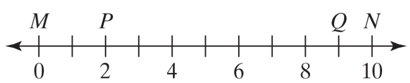
Mini-Lesson 10.7
Geometric Probability

Learning Objectives:

1. Use segment and area models to find the probabilities of events.
2. Key vocabulary: *geometric probability*

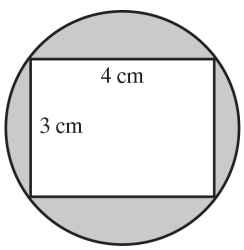
Key Examples:

1. Point J on $\overline{MN}$ is chosen at random. What is the probability that J lies on $\overline{PQ}$?

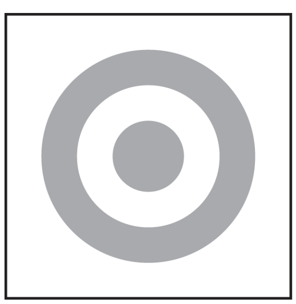


2. A city bus stops at the southeast corner of Michigan Avenue and Huron Street in Chicago every 12 minutes. If a passenger arrives at the bus stop at a random time, what is the probability that the passenger will wait at least 3 minutes for the bus?

3. A rectangle is inscribed in a circle. Point P in the circle is selected at random. What is the probability that the point lies in the shaded region?



4. Assume that a dart thrown at a 12-in.-by-12-in. square dartboard is equally likely to land at any point on the board. The diameter of the bullseye (center circle) is 3 in., and the width of each ring is 1.5 in. What is the probability of hitting the bullseye?



Answers: 1) $\frac{7}{10}$, or 0.7, or 70% 2) $\frac{3}{4}$, or 0.75, or 75% 3) 0.389, or 38.9% 4) about 0.049, or about 4.9%

Mini-Lesson 10.7

Geometric Probability

MyMathLab®

Geometry
Multimedia Lesson Resources

eText, Section 10.7 (MML)

Interactive Lecture Video Section 10.7

Interactive Lecture Video Objective 1

Video Organizer Section 10.7 (print)

Video Organizer Section 10.7 (MML)

PowerPoints, Section 10.7

Teaching Notes:

- Begin this section with a review of the basic vocabulary of probability: *sample space, outcome, event.* Explain that in problems involving geometric probability, the *sample space* will be the set of all points under consideration (such as the points on a line segment or in the interior of a circle), an *outcome* will be specific point, and an *event* will be the set of all points in the sample space satisfying a given condition.

ERROR PREVENTION

- Some students may make errors because they fail to calculate probability as a ratio or reverse the numerator and denominator. Review the definition of the theoretical probability of an event given at the top of p.471 in the textbook, hiliting the words *favorable* and *possible.* Also remind them that the probability of an event can never be greater than 1.

Closure Questions:

- How are geometric probability problems like other probability problems you have solved? How are they different?

  They are alike because they use the same basic vocabulary and definition of the probability of an event. They are different because in geometric probability, the sample space is a set of points rather than a set of numbers, letters, or other objects.