Chapter 12: Areas and Volumes of Solids  
Unit 1: Important Solids  
Section 1: Prisms

**Definition**
A **prism** is a solid with two congruent bases that are parallel, and lateral edges are parallel.

Some Types of Prisms

What is the difference between right prism and oblique prism?

**Theorem 12.1**
The lateral area of a right prism equals the perimeter, \( p \), of a base times the height, \( h \), of the prism. \( (L.A. = ph) \)

**Theorem 12.2**
The volume of a right prism equals the area of a base, \( B \), times the height, \( h \), of the prism. \( (V = Bh) \)
Example 5
Given that the volume of a cube is 8 cm$^3$, find
(a) the length of each edge.
(b) total surface area. Show in two different way.

Example 6
The length of a rectangular prism is four times the width and the height equal to the width. If the volume is 32m$^3$, then find dimension.
**Example 7**

A drinking trough for horses is a right trapezoidal prism with dimensions shown. If the water is filled 2/3 of the way up, about how will the water weight? (Note: 1 m$^3$ of water weight 1 metric ton).

We'll be studying mostly regular pyramid in this section. **Regular pyramids have following properties:**

- the base is a regular polygon
- all lateral edges are congruent
- all lateral faces are isosceles triangle
- the altitude meets the base at its center
Given regular pentagonal pyramid, name the following

- base:
- lateral edges:
- lateral faces:
- altitude:
- slant height:
- apex:

Theorem 12.3
The lateral area of a regular pyramid equals half the perimeter of the base times the slant height. \( \text{L.A.}=\frac{pl}{2} \)

Can you prove the formula of lateral area?

Theorem 12.4
The volume of a pyramid equals one third the area of the base times the height of the pyramid. \( \text{Volume} = \frac{Bh}{3} \)
Chapter 12: Areas and Volumes of Solids
Unit 1: Important Solids
Section 2: Pyramids

Example 1
A regular square pyramid has base edge 6m and lateral edge 5m.

(a) length of a slant height = ____________
(b) lateral area = ____________
(c) base area = ____________
(d) total area = ____________
(e) length of altitude = ____________
(f) volume = ____________

Example 2
A regular triangular pyramid has slant height 9cm and base perimeter 12cm.

(a) lateral area = ____________
(b) base area = ____________
(c) total area = ____________

Example 3
The height of a regular triangular pyramid is 6cm, and the radius of the base is 8cm. Find the base area and the volume of the pyramid.

Example 4
Find the volume of the regular pyramid below to the nearest tenth. The measure of each side of the base is 6.
Chapter 12: Areas and Volumes of Solids
Unit 1: Important Solids
Section 3: Cylinders and Cones

on your desk

Definition
Cylinder is a prism with bases that are congruent circles on a parallel plane.

Cones is a pyramid with a circular base. All formulae shown in previous two sections apply to both cylinder and cones.

COMPARE AND CONTRAST the new theorems from Thm12.1 & Thm12.3 !!!!

Theorem 12.1
The lateral area of a right prism equals the perimeter, p, of a base times the height, h, of the prism. \( (L.A. = ph) \)

Theorem 12.2
The volume of a right prism equals the area of a base, B, times the height, h, of the prism. \( (V = Bh) \)

Theorem 12.5
The lateral area of a cylinder equals the circumference of a base, \( 2\pi r \), times the base times the height, h, of the cylinder. \( (L.A. = 2\pi rh) \)

Theorem 12.3
The lateral area of a regular pyramid equals half the perimeter of the base times the slant height. \( (L.A. = pl/2) \)

Theorem 12.7
The lateral area of a cone equals half the circumference of the base times the slant height. \( (L.A. = \pi rl/2 = \pi rl) \)

Theorem 12.4
The volume of a pyramid equals one third the area of the base times the height of the pyramid. \( (Volume = Bh/3) \)

Theorem 12.8
The volume of a cone equals one third the area of the base times the height of the pyramid. \( (Volume = Bh/3 = \pi r^2 h/3) \)
Example 5
A cone with radius 6cm and height 12cm is filled to capacity with liquid. Find the minimum height of a cylinder with radius 4cm that will hold the same amount of liquid.

Example 6
A right cone and a right cylinder have equal base areas. The height of the cylinder is four times the height of the cone. Compare the volumes.

Example 7
A pipe is 3m long and has inside radius 4cm and outside radius 5cm. Find the volume of metal. Round to the nearest tenth. Draw the diagram, first.
### Example 8

A regular square pyramid is inscribed in a cone with radius 4cm and height 4cm.

(a) What is the volume of the pyramid?

(b) Find the slant height of the cone and the pyramid.

![Diagram of a square pyramid inscribed in a cone](image)

---

### Theorem 12.9

The surface area of a sphere is \(4\pi r^2\)

### Theorem 12.10

The volume of a sphere is \(\frac{4}{3}\pi r^3\)

<table>
<thead>
<tr>
<th></th>
<th>d</th>
<th>r</th>
<th>A</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>32000\pi</td>
<td>(\frac{3}{3})</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>12\pi</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>3r</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>2r</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 7
The volume of a sphere is $36\pi$. Find the surface area.

Example 8

a) Compare the areas of the spheres in 5 and 6.

b) Compare the volumes of the spheres in 5 and 6.

c) What can you generalize from this example?

Example 10
The number of square centimeters in the area of a sphere is twice the number of cubic centimeter in the volume of a sphere. Find the radius of the sphere.

Example 11
A scoop of ice cream with diameter 8cm is placed on an ice-cream cone with diameter 7cm and height 10cm. If the ice creams melts, will the ice cream overflow? Show why or why not.

Example 12
Four metal ball with a diameter of 4cm fit snugly inside a cylinder. If you melt it and pour it back into the cylinder, how high will the molten metal be after it cools inside the cylinder?
Similar solids are solids that have the same shape, but not necessarily the same size. It’s analogous to similar polygons.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>II</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>radius = 2</td>
<td>radius = 4</td>
<td>side of base = 3</td>
<td>side of base = 9</td>
</tr>
<tr>
<td>height = 6</td>
<td>height = 12</td>
<td>height = 4</td>
<td>height = 12</td>
</tr>
</tbody>
</table>

Find the ratios of volumes for each set of solids.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>radius = 2</td>
<td>radius = 4</td>
<td>side of base = 3</td>
<td>side of base = 9</td>
</tr>
<tr>
<td>height = 6</td>
<td>height = 12</td>
<td>height = 4</td>
<td>height = 12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Cylinder I and II</th>
<th>Pyramids I and II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>scale factor</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>base perimeter I</td>
<td></td>
</tr>
<tr>
<td></td>
<td>base perimeter II</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>L.A. I</td>
<td></td>
</tr>
<tr>
<td></td>
<td>L.A. II</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Volume I</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Volume II</td>
<td></td>
</tr>
</tbody>
</table>
Theorem 12.11
If the scale factor of two similar solids is $a:b$, then
12.1 (1) the ratio of corresponding perimeters is $a:b$.
12.2 (2) the ratio of the base areas, the lateral areas, and the total area is $a^2:b^2$.
12.3 (3) the ratio of the volume is $a^3:b^3$.

Example 1
(a) Are all spheres similar? (b) Are all right cylinders similar?

Example 2
Two cones have base radii 8 and 12, and heights 16 and 24, respectively. Are the cones similar?

Example 3
Two similar square pyramids have base areas 4m$^2$ and 36m$^2$.
- Find the ratio of the height of the pyramids.
- If the height of the larger pyramid is 27m, what is the height of the smaller pyramid?

Problem 1
Two regular pyramids have equilateral triangular bases with sides 4 and 6. Their heights are 6 and 9, respectively. Are the two pyramids similar?

Problem 2
Two similar cones have bases with area ratios of 4:9. Find the ratios of the following:
(a) radii (b) heights (d) total areas (d) volumes

Problem 3
The volumes of two spheres have a ratio of 24:64. Find the area of the larger sphere if the area of the smaller sphere is 18.

Problem 4
The radii of two similar cylinders are 2 and 5. Find the ratios of their volumes and of their lateral areas.
<table>
<thead>
<tr>
<th>Problem 5</th>
<th>The volumes of two similar rectangular solids are $125\text{cm}^3$ and $64\text{cm}^3$. Find the ratio of their base perimeters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 6</td>
<td>(a) A diagonal of one cube is 2cm. Find its volume.</td>
</tr>
<tr>
<td></td>
<td>(b) Using the ratios of similar solids, find the volume of a cube whose diagonal is 5cm.</td>
</tr>
</tbody>
</table>