Chapter 13: Coordinate Geometry

Unit 1: Geometry and Algebra

Section 1: The Distance Formula

Let’s review some basic terms you learned in Algebra.

Coordinate plane shown is referred as “x-y coordinate plane”, “rectangular coordinate plane”, or “Cartesian coordinate plane.”

Theorem 13.1 The Distance Formula

The distance between points \((x_1, y_1)\) and \((x_2, y_2)\) is given by:

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Example 1

Find the distance between the origin and the point names.

(a) \((0, -6)\)  (b) \((8, 0)\)  (c) \((7, 24)\)

Example 2

Find the distance between the points. Write answer in simplest form.

(a) \((2, 5), (8, 5)\)  (b) \((-3, -8), (2, 4)\)

(c) \((6, 0), (5, 1)\)  (e) \((9, 4), (1, -8)\)
Theorem 13.2 The Equation of the Circle

An equation of the circle with center \((a,b)\) and radius \(r\) is

\[(x - h)^2 + (y - k)^2 = r^2\]

**Example 3**

Find the equation of a circle with the given center and radius.

(a) \((4,-3), r=5\)  
(b) \((-1,-2), r = \sqrt{5}\)

**Example 2**

Find the center and radius of each circle.

(a) \((x - 8)^2 + (y - 1)^2 = 64\)  
(b) \((x + 9)^2 + (y - 7)^2 = 25\)
Chapter 13: Coordinate Geometry
Unit 1: Geometry and Algebra
Section 5: The Midpoint Formula

Theorem 13.5 The Midpoint Formula
The midpoint of the segment that joins points \((x_1, y_1)\) and \((x_2, y_2)\) is the point \(
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\)

Example 1
Find the midpoint of segment AB.
(a) \(A(6,3)\), \(B(-2,-5)\)  
(b) \(A(a,b)\), \(B(c,b)\)

Example 2
\(M (4,-2)\) is the midpoint of the segment AB. If A has coordinates \((2,-5)\), find the coordinate of B.  
\((x+9)^2 + (y-7)^2 = 25\)
Slope of a line indicates its steepness. See stairs, for example.

Type of slopes:

- $m = \text{undefined}$
- $m = 0$
- $m < 0$
- $m > 0$
Chapter 13: Coordinate Geometry
Unit 1: Geometry and Algebra
Section 2: Slope of a Line

Example 1
Always, sometimes, or never.
a) The slope of a vertical line is _________ zero.
b) The slope of a horizontal line is _________ zero.
c) The slope of a line that rises to the right is _________ positive.
d) The slope of a line that falls to the right is _________ negative.

Example 2
Find the slope of the line through the two points names.
1. (-3,4), (-4,5)
2. (6, -3), (-1, -2)
3. (8, -4), (-3, -4)
4. (-6, -2), (-6, 9)

Notes
Chapter 13: Coordinate Geometry
Unit 1: Geometry and Algebra
Section 3: Parallel and Perpendicular Lines

Theorem 13.3
Two non-vertical lines are parallel if and only if their slopes are equal.
What can you conclude about two vertical lines?

Theorem 13.4
Two non-vertical lines are perpendicular if and only if the product of their slope is \(-1\).
What can you conclude about a horizontal line and a vertical line?
### Example 1
Complete the table below for $r \parallel s$ and $r \perp t$.

<table>
<thead>
<tr>
<th>slope of $r$</th>
<th>slope of $s$</th>
<th>slope of $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$-1/2$</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td></td>
<td>$1/6$</td>
</tr>
<tr>
<td>$c$</td>
<td></td>
<td>$1/5$</td>
</tr>
<tr>
<td>$d$</td>
<td></td>
<td>$-4$</td>
</tr>
<tr>
<td>$e$</td>
<td></td>
<td>$a$</td>
</tr>
<tr>
<td>$f$</td>
<td></td>
<td>$3b/4$</td>
</tr>
</tbody>
</table>

### Example 2
If $r \parallel s$ and $r$ is a horizontal line, what is the slope of $s$?

### Example 3
If $k \perp n$, and $n$ has slope 0, what is the slope of $k$?

### Example 4
Use slopes to show that a quadrilateral with vertices $A(-2,7)$, $B(3,7)$, $C(6,11)$, and $D(1,11)$ is a parallelogram.

### Example 5
Plot points $A(4,0)$, $B(0,-8)$, and $C(-16,0)$. Classify the triangle $ABC$. 


**Theorem 13.6: Standard Form (otherwise known as General Form)**

The graph of any equation that can be written in the form

\[ Ax + By = C \]

where \( A \) and \( B \) are not both zero, is a line.

---

**Theorem 13.7: Slope-Intercept Form**

A line with the equation \( y = mx + b \) has slope \( m \) and y-intercept \( b \).
Example 4
Use slopes to show that a quadrilateral with vertices A(−2,7), B(3,7), C (6,11), and D(1,11) is a parallelogram.

Example 5
Plot points A(4,0), B(0,−8), and C(−16,0). Classify the triangle ABC.