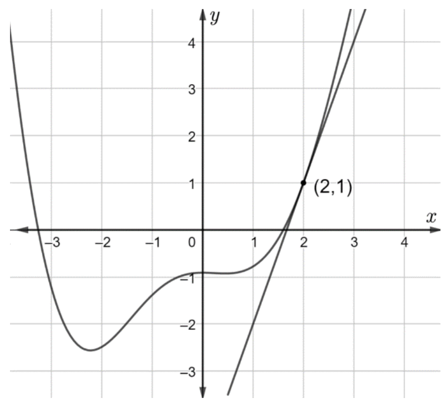
AP CALCULUS BC	YouTube Live Virtual Lessons	Mr. Bryan Passwater Mr. Anthony Record
Topic: 10.11	**Finding Taylor Polynomial Approximations of Functions – Day 2**	**Date:** April 9, 2020

Warm-Up

AP Practice Problem



A function f has derivatives of all orders for all values of x. A portion of the graph of f is shown above with the line tangent to the graph of f at $x = 2$. Let g be the function defined by $g(x) = 3 + \int_2^x f(t)\,dt$.

a.) Find the second degree Taylor polynomial for $g(x)$ centered at $x = 2$.

b.) Does $g(x)$ have a local minimum, local maximum, or neither at $x = 2$? Give a reason for your answer.

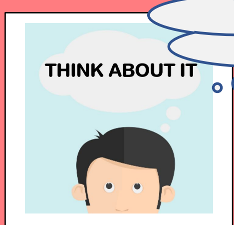
c.) Consider the geometric series $\sum_{n=1}^{\infty} a_n$ where $a_1 = g'(2)$ and $a_2 = g'(x) - 1$. Find $\sum_{n=1}^{\infty} a_n$ when $x = \dfrac{13}{6}$.

Lesson Overview

WHAT WE ARE GOING TO DO	WHAT YOU SHOULD ALREADY KNOW
• **Continue investigating the importance of Taylor polynomials and the role they play in the mathematics world.** • **Solve unique conceptual-based problems with Taylor polynomials.**	• **How to develop a Taylor polynomial from a given function.** • **The difference between a Tylor polynomial and a Maclaurin polynomial.** • **The components that make up the coefficients of the terms in a Taylor polynomial.**

WHAT YOU WILL BE ABLE TO DO

Let *f* be a function having derivatives for all orders of real numbers. The 4th degree Taylor polynomial for *f* about $x = -2$ is given by $P_4(x) = -8 - \frac{5}{2}(x+2)^2 + k(x+2)^4$, where $k \neq 0$.

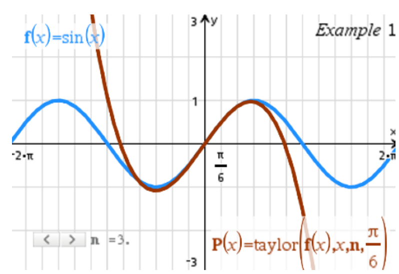


Question: If $f^{(4)}(-2) = \frac{2\pi}{3}$, find the value of *k*.

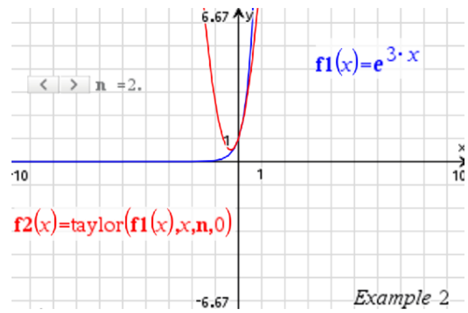
Guided Practice

Example 1: Find the third degree Taylor polynomial for $f(x) = \sin x$ centered at $x = \frac{\pi}{6}$.

Use this polynomial to approximate $\sin(0.2)$.



Example 2: Find the second degree Maclaurin polynomial for $g(x) = e^{3x}$.



x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$
3	1	-2	7	4	-5

Example 3: The functions f and g are differentiable for all orders. The values of f, g, and selected derivatives of each are given in the table above at $x = 3$. For $n \ge 2$, the nth derivative of g at $x = 3$ is given by $g^{(n)}(3) = f^{(n-2)}(3)$. Find the third degree Taylor polynomial for $g(x)$ about $x = 3$.

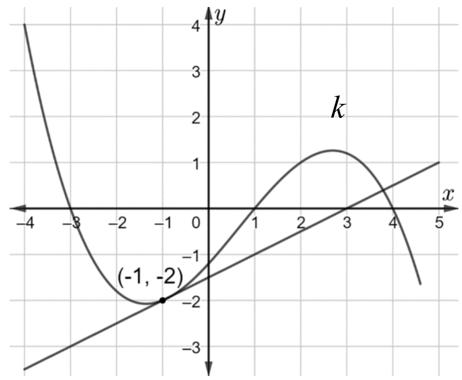
Example 4: A function $f(x)$ is not explicitly known but it is known that $f(2) = -7$ and $f'(2) = 0$. Additionally, for $n > 1$, $f^{(n)}(2) = \dfrac{n-1}{3^n}$. Find a 4th degree Taylor polynomial for $f(x)$ centered at $x = 2$. Use this polynomial to approximate $f(3)$.

x	1	2
$f(x)$	–2	0
$f'(x)$	3	–1
$f''(x)$	5	–6

Example 5: The function h is defined by $h(x) = 4 + \int_{2}^{2x} f(t)\,dt$ where f is a twice differentiable function.

Selected values of f and its derivatives are given in the table above. Find the 2nd degree Taylor polynomial for $h(x)$ centered at $x = 1$.

Check for Understanding



Practice 1: A function k has derivatives of all orders for all values of x. A portion of the graph of k is shown above with the line tangent to the graph of k at $x = -1$. For $n \geq 2$, the nth derivative of $k(x)$ at $x = -1$ is given by:

$$k^{(n)}(-1) = \frac{n!}{n+1}.$$

Find the third degree Taylor polynomial for $k(x)$ about $c = -1$.

Practice 2: The functions f, f', and f'' are each continuous and differentiable. The nth derivative of f is given by $f^{(n)}(1) = \sum_{i=0}^{\infty} 12\left(\frac{n+1}{5}\right)^i$ when $0 \leq n \leq 3$. Find the third degree Taylor polynomial for $f(x)$ centered around $x = 1$.

Practice 3: The function g is continuous and has derivatives for all orders at $x=-1$. It is known that $g(-1)=7$ and for positive values of n, the nth derivative of g at $x=-1$ is defined as the piecewise function given below:

$$g^{(n)}(-1)=\begin{cases} n^2+1, & n \text{ is odd} \\ 0, & n \text{ is even} \end{cases}$$

a.) Find $P_5(x)$, the fifth degree Taylor polynomial of $g(x)$ centered at $x=-1$.

b.) Determine if $P_5(x)$ is increasing or decreasing at $x=-1$. Explain your reasoning.

Practice 4: The fourth degree Taylor polynomial for $f(x)$ centered about $x=2$ is given by

$T_4(x)=2-3(x-2)+\dfrac{3(x-2)^2}{4}-\dfrac{4(x-2)^3}{9}+\dfrac{7(x-2)^4}{26}$. Find the value of $f'''(2)$.

Practice 5: Consider the function $g(x) = e^{\frac{-2x}{3}}$. Find the coefficient of the x^{26} term in the Maclaurin polynomial of degree 26.

Debrief and Summary

ENDURING UNDERSTANDINGS	KEY TAKEAWAYS
Power series allow us to represent associated function on an appropriate interval.	• A Taylor polynomial allows us to approximate the values of more complicated function. • In many cases, as the degree of the Taylor polynomial increases, the polynomial will approach the original function over some growing interval. • Taylor polynomials can be extended to Taylor series and leads to the idea of power series.

COMMON ERRORS, MISCONCEPTIONS & PITFALLS

- Students sometimes forget that the coefficients of the terms of a Taylor polynomial are of the form $\dfrac{f^{(n)}(c)}{n!}$.
- Taylor polynomials can produce approximations, but we have not learned how “good” that are at approximating our desired function….yet!

AP Exam Practice

AP Practice Problem

A function f has derivatives of all orders at all real x values.

a.) Let $P_2(x)$ represent the 2nd degree Maclaurin polynomial for f. It is known that $f(0)=1$ and $f'(0)=0$. If $P_2(1)=\frac{1}{2}$, find $f''(0)$.

b.) Find $P_2'(0)$ and $P_2''(0)$. Does $P_2(x)$ have a relative minimum, relative maximum, or neither at $x = 0$? Give a reason for your answer.

c.) Use $P_2(x)$ to approximate $f\left(\frac{1}{2}\right)$.