AP CALCULUS BC	YouTube Live Virtual Lessons	Mr. Bryan Passwater Mr. Anthony Record
Topic: 10.7	**Alternating Series Test for Convergence**	**Date:** March 31, 2020

Warm-Up

Determine, if possible, if each of the given series diverges based on the nth term test for divergence

a) $\sum_{n=0}^{\infty} 5^n$

b) $\sum_{n=0}^{\infty} \frac{2n!}{1-3n!}$

c) $\sum_{n=0}^{\infty} (3^{1-n} \cdot 2^{1+n})$

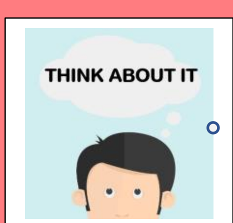
Lesson Overview

WHAT WE ARE GOING TO DO	WHAT YOU SHOULD ALREADY KNOW
• Solidify our understanding of partial sums and convergence vs. divergence of a series • Understand and use the Alternating Series Test when applicable	• The nth term test for divergence • How to evaluate a limit at infinity

WHAT YOU WILL BE ABLE TO DO

Consider the four series given below. Which of these series can be shown to converge using the alternating series test?

$$I.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \qquad II.\ \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot n}{2n+3} \qquad III.\ \sum_{n=0}^{\infty} \frac{\cos(\pi n)}{n!} \qquad IV.\ \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n+2}$$



Question: Which of these meet the conditions needed to apply the alternating series test?

Guided Practice

Let's briefly review some of the important concepts and vocabulary needed for Unit 10…

Sequence a_n	Series $\sum_{n=1}^{\infty} a_n$
Partial Sum: $S_n = a_1 + a_2 + a_3 + \cdots + a_n$	
A series converges if: $\lim_{n\to\infty} S_n = L$	A series diverges if: $\lim_{n\to\infty} S_n \neq L$
n-th term test for divergence If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges	

Given a series, we are interested in knowing if the series converges or diverges. If a series converges, we are often interested in what the series converges to…although this is not always possible (especially in an entry level calculus course).

Example 1: Consider the following series below. Determine if the series converge or diverge.

$a)\ \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{2^n} + \cdots$

$b)\ \sum_{n=1}^{\infty} (-1)^n$

$c)\ \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

Alternating Series

An alternating series is a series whose terms alternate in signs.

A few examples of alternating series include:

a) $\frac{7}{3} - \frac{1}{2} + 2 - \frac{1}{4} + \frac{5}{3} - \frac{1}{8} + \frac{4}{3} - \frac{1}{16} + \cdots$

b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$

c) $\sum_{n=0}^{\infty} \left(\frac{-2}{3}\right)^n$

d) $\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{n!}$

Alternating Series Test

Let $a_n > 0$. The alternating series

$$\sum_{n=1}^{\infty} (-1)^n a_n \quad \text{and} \quad \sum_{n=1}^{\infty} (-1)^{n-1} a_n$$

converge if the following conditions are both met:

1. $\lim_{n\to\infty} a_n = 0$
2. $a_{n+1} \le a_n$ for all $n > N$ where N is an integer

Example 2: Determine the convergence or divergence of $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

Example 3: Determine the convergence or divergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 - 6n + 10}$

Example 4: Consider the series given below. For each, determine if the altnernating series test can be applied. If not, explain why not.

$a)\ \sum_{n=0}^{\infty} \frac{\cos(\pi n)}{n!}$

$b)\ \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{5n - 3}$

$c)\ \sum_{n=0}^{\infty} \frac{(-1)^{\mathrm{n}+1} \cdot n}{3n + 1}$

$d)\ \sum_{n=1}^{\infty} a_n \text{ where } a_n = \begin{cases} \frac{1}{n}, & \text{if } n \text{ is odd} \\ -\frac{1}{n^2}, & \text{if } n \text{ is even} \end{cases}$

Check for Understanding

Practice 1: Show that the following series converges using the alternating series test

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} = 1 - \frac{1}{2} + \frac{1}{24} - \frac{1}{720} + \cdots$$

Practice 2: There are six series listed below. For each of the six series, determine which of the two categories below they fall into.

Alternating Series Test does not apply	**Converges by the Altnerating Series Test**

A. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2}$	B. $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{2n-1}{7n+3}$
C. $\sum_{n=0}^{\infty} \frac{(-1)^{2n}}{4^n}$	D. $\sum_{n=1}^{\infty} \cos(\pi n) \cdot n^{-1}$
E. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{n^2+2}$	F. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt[3]{n}}{n}$

Practice 3: Each statement below if false. Correct each statement to create a true statement.

For Statements $1-3$: Let $a_n > 0$

Statement 1: If $a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n$ converges, then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges

Statement 2: If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges

Statement 3: If $\sum_{n=1}^{\infty} (-1)^n a_n$ diverges, then $\lim_{n\to\infty} a_n = 0$

Statement 4: Consider the series $\sum_{n=1}^{\infty} b_n$. If $\sum_{n=1}^{\infty} b_n$ diverges, then $\lim_{n\to\infty} b_n \neq 0$

Debrief and Summary

ENDURING UNDERSTANDING	KEY TAKEAWAY
Applying limits may allow us to determine the finite sum of infinitely many terms.	For $a_n > 0$, the alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ converges if 1. $\lim_{n\to\infty} a_n = 0$ 2. $a_{n+1} \le a_n$ for all $n > N$ where N is an integer

COMMON ERRORS, MISCONCEPTIONS & PITFALLS

- The alternating series test is for CONVERGENCE.
- For any series, if $\lim_{n\to\infty} a_n \neq 0$, the series diverges
- A divergent series does **not** necessarily imply $\lim_{n\to\infty} S_n \to \infty$

AP Exam Practice

Consider the altnerating series defined below:

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

A) Use the alternating series test to show that this series converges when $x = 3$.

B) Show that this series converges for all x values where x is a real number.

C) Consider the function $f(x)$ where $f'(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

Determine if $f(x)$ has a relative minimum, relative maximum or neither at $x = 0$.
Give a reason for your answer.