Calculus Section 9.1 Sequences

Homework: page 596 #'s 1 – 7 odd, 18, 19, 29 – 49 odd

-List the terms of a sequence and write a sequence.
-Determine whether a sequence converges or diverges

Mathematically, a **sequence** is defined as a function whose domain is the set of positive integers. Each integer is mapped to a term of the sequence.

$$\begin{array}{cccccccc} 1, & 2, & 3, & 4, & 5, & \dots & n, & \dots \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ a_1 & a_2 & a_3 & a_4 & a_5 & \dots & a_n & \dots \end{array}$$

The numbers $a_1, a_2, a_3, \dots, a_n, \dots$ are the **terms** of the sequence. The number a_n is called the **nth term** of the sequence, and the entire sequence is notated using curly-brackets: $\{a_n\}$.

Example) List the terms of each sequence:

1) $\{a_n\} = \{3 + (-1)^n\}$

2) $\{a_n\} = \left\{\frac{1}{2^n}\right\}$

3) $\{d_n\}$ is $d_{n+1} = d_n - 5$, $d_1 = 25$

A primary focus of this chapter concerns sequences whose terms approach limiting values. These sequences are said to **converge**. For instance, the sequence $\left\{\frac{1}{2^n}\right\}$ converges to 0.

Evaluate $\lim\limits_{n\to\infty} \{a_n\}$ to determine whether (and to what) a sequence converges.

Example) Determine whether each sequence converges

1) $\{a_n\} = \{3 + (-1)^n\}$

2) $\{b_n\} = \left\{\dfrac{n}{1-2n}\right\}$

3) $\{c_n\} = \dfrac{n^2}{2^n - 1}$

4) $\{a_n\} = \left\{\dfrac{\ln(n)}{n}\right\}$

5) $\{b_n\} = \dfrac{(n+1)!}{n!}$

6) $\{c_n\} = \dfrac{(n+1)!}{(n+3)!}$

Properties of Limits of Sequences

Let $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = K$.

1) $\lim_{n\to\infty}(a_n \pm b_n) = L \pm K$

2) $\lim_{n\to\infty}(ca_n) = cL$, c is any real number

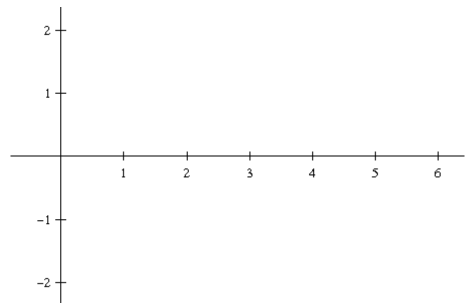
3) $\lim_{n\to\infty}(a_n b_n) = LK$

4) $\lim_{n\to\infty}(\frac{a_n}{b_n}) = \frac{L}{K}$, $b_n \neq 0$ and $K \neq 0$

Squeeze Theorem for Sequences

If $\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} b_n$ and there exists an integer N such that

$a_n \le c_n \le b_n$ for all $n > N$, then $\lim_{n\to\infty} c_n = L$.



Example) Show that the sequence $\{c_n\} = \left\{(-1)^n \frac{1}{n!}\right\}$ converges, and find its limit.

Find the nth Term of a Sequence

Find a sequence $\{a_n\}$ whose first five terms are $\frac{-2}{1}, \frac{8}{2}, \frac{-26}{6}, \frac{80}{24}, \frac{-242}{120}$ … then determine the value of a_6 and whether the sequence converges or diverges.