

Summary of Tests for Convergence of Infinite Series

<i>n</i> -th Term Test	$\sum_{n=1}^{\infty} a_n \text{ converges} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0.$ $\lim_{n \rightarrow \infty} a_n \neq 0 \Rightarrow \sum_{n=1}^{\infty} a_n \text{ diverges.}$
Geometric series	$\sum_{i=1}^{\infty} ar^{i-1} \text{ converges if and only if } r < 1. \text{ If the series converges,}$ <p>its sum is $\frac{a}{1-r}$</p>
Integral Test	$f(x)$ is continuous, positive, and decreasing. $\sum_{n=1}^{\infty} f(n) \text{ converges} \Leftrightarrow \int_M^{\infty} f(x) dx \text{ converges (for some } M).$
<i>p</i> -Series	$\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ converges} \Leftrightarrow p > 1.$
Comparison Test	$0 < a_n < b_n.$ $\sum_{n=1}^{\infty} b_n \text{ converges} \Rightarrow \sum_{n=1}^{\infty} a_n \text{ converges.}$ $\sum_{n=1}^{\infty} a_n \text{ diverges} \Rightarrow \sum_{n=1}^{\infty} b_n \text{ diverges.}$
Limit Comparison Test	$a_n > 0$ and $b_n > 0$ $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} > 0 \left(\sum_{n=1}^{\infty} a_n \text{ converges} \Leftrightarrow \sum_{n=1}^{\infty} b_n \text{ converges} \right)$ $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0 \text{ and } \sum_{n=1}^{\infty} b_n \text{ converges} \Rightarrow \sum_{n=1}^{\infty} a_n \text{ converges}$ $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty \text{ and } \sum_{n=1}^{\infty} b_n \text{ diverges} \Rightarrow \sum_{n=1}^{\infty} a_n \text{ diverges}$
Ratio Test	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right < 1 \Rightarrow \sum_{n=1}^{\infty} a_n \text{ converges.}$ $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right > 1 \Rightarrow \sum_{n=1}^{\infty} a_n \text{ diverges.}$ $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 1 \Rightarrow \text{can't tell.}$
Alternating Series Test	$a_n > 0$, decreasing, $\lim_{n \rightarrow \infty} a_n = 0 \Rightarrow \sum_{n=1}^{\infty} (-1)^{n-1} a_n \text{ converges.}$

An additional test for convergence is the root test, but this is not tested on AP Exams.