UNIT 7 REVIEW: Differential Equations **AP Calc AB April 22, 2020**

Note: Videos from March 25th – April 3rd cover the entire Unit 7. These are selected highlights.

Example 1: The rate of change of the height of a tree, h, in meters, with respect to the age of the tree, t, in years, is inversely proportional to the product of the time and the cube root of the height. Write this situation as a differential equation. $\frac{dh}{dt} = \frac{k}{t\cdot\sqrt[3]{h}} = \frac{k}{\sqrt[3]{h}\cdot t} = \frac{k}{t\cdot h^{1/3}}$

Example 2: For what value of k, if any, will $y = e^{2x} + ke^{-5x}$ be the solution to the differential equation $4y - y'' = 30e^{-5x}$?

$y' = 2e^{2x} - 5ke^{-5x}$; $\quad y'' = 4e^{2x} + 25ke^{-5x}$

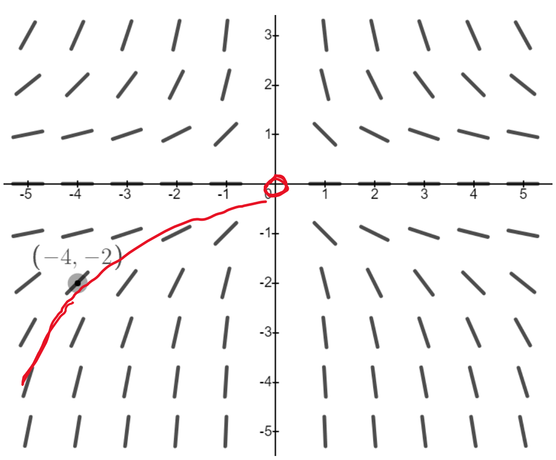
$$4(e^{2x} + ke^{-5x}) - (4e^{2x} + 25ke^{-5x}) = 30e^{-5x}$$

$$\cancel{4e^{2x}} + 4ke^{-5x} \cancel{- 4e^{2x}} - 25ke^{-5x} = 30e^{-5x}$$

$$-21k = 30$$

$$k = -\frac{10}{7}$$

Example 3: Consider the differential equation $\frac{dy}{dx} = -\frac{y^2}{x}$, $x \neq 0$.
Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(-4) = -2$.



(a) Sketch the solution curve to through the point $(-4, -2)$.

(b) Write the equation of the tangent line to the solution curve at the point $(-4, -2)$. $m = \left.\frac{dy}{dx}\right|_{(-4,-2)} = -\frac{(-2)^2}{-4} = 1$

$$y = -2 + 1(x - (-4))$$

(c) Use the equation of the tangent line to approximate $f(-4.1)$.

$$f(-4.1) \approx -2 + (-4.1 + 4) = -2.1$$

(d) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(-4) = -2$.

$$\int \frac{dy}{y^2} = \int -\frac{1}{x}dx$$

$$-y^{-1} = -\ln|x| + C$$

$$-\frac{1}{(-2)} = -\ln|-4| + C$$

$$C = \frac{1}{2} + \ln 4$$

$$-y = \frac{1}{-\ln|x| + \frac{1}{2} + \ln 4}$$

$$\boxed{y = \frac{-1}{-\ln|x| + \frac{1}{2} + \ln 4}} = \frac{-2}{1 - \ln\left|\frac{x}{4}\right|}$$

Example 4: Given that $\frac{dG}{d\theta} = \frac{\theta \sin(\theta^2)}{G}$ and that $G\left(\sqrt{\frac{\pi}{3}}\right) = -2$, determine $G\left(\sqrt{\frac{\pi}{2}}\right)$.

$$\int G\,dG = \frac{1}{2}\int 2\theta \sin(\theta^2)\,d\theta$$

$$\frac{1}{2}G^2 = -\frac{1}{2}\cos(\theta^2) + C$$

$$\frac{1}{2}(-2)^2 = -\frac{1}{2}\cos\left(\sqrt{\frac{\pi}{3}}^2\right) + C$$

$$\frac{9}{4} = C$$

$$G(\theta) = -\sqrt{-\frac{1}{2}\cos(\theta^2) + \frac{9}{2}}$$

$$G\left(\sqrt{\frac{\pi}{2}}\right) = \sqrt{4.5}$$

Example 5: The point $(1, 2)$ is on the graph of the solution curve to the differential equation $\frac{dy}{dx} = (x+2)(3-y)$. Find the y-coordinate such that the point $(2, y)$ is also on the graph of the solution curve.

$$\frac{dy}{3-y} = (x+2)dx$$

$$\int \frac{dy}{3-y} = \int (x+2)\,dx$$

$$-\ln|3-y| = \frac{1}{2}x^2 + 2x + C$$

$$C = -2.5$$

$$@(2, y): -\ln|3-y| = 2 + 4 - 2.5$$

$$|3-y| = e^{-(3.5)}$$

$$-y = -3 \pm e^{-3.5}$$

$$y = 3 \mp e^{-3.5} \quad \text{(which form for } (1,2)?)$$

$$y = 3 - e^{-3.5}$$

Example 6: In a certain locale, there are 2345 confirmed cases of a virus. The number of confirmed cases at time t, in days, is given by $N(t)$. The rate at which the number of confirmed cases is changing can be modeled by the differential equation $\frac{dN}{dt} = 0.336N$. Determine an equation for $N(t)$.

$$N(t) = 2345e^{0.336t}$$