AP CALCULUS BC	YouTube Live Virtual Lessons	Mr. Bryan Passwater Mr. Anthony Record
Topic: All Units	**Free Response Question Stem Types** **Tabular**	**Date:** April 27, 2020

Free Response Questions Stem Types: Tabular

2020 FRQ Practice Problem BC1

x	1	2	3	4	5
$f'(x)$	62	30	20	15	12

BC1: The function f is twice differentiable for $x \geq 1$ where $f(5) = -6$. Selected values of the positive and decreasing function f', the derivative of f, are given in the table above. The graph of f' has horizontal asymptote $y = 0$.

The series $\sum_{n=1}^{\infty} a_n$ is defined where $a_n = f'(n)$.

(**a**) Use a right Riemann sum with the four subintervals indicated in the table to approximate $f(1)$. Is this approximation an over or under estimate of $f(1)$? Give a reason for your answer.

(**b**) Evaluate $\int_1^{\infty} f''(x)dx$.

(**c**) If $\int_1^5 xf''(x)dx = -100$, find $f(1)$.

The problem has been restated.

x	1	2	3	4	5
$f'(x)$	62	30	20	15	12

BC1: The function f is twice differentiable for $x \geq 1$ where $f(5) = -6$. Selected values of the positive and decreasing function f', the derivative of f, are given in the table above. The graph of f' has horizontal asymptote $y = 0$.

The series $\displaystyle\sum_{n=1}^{\infty} a_n$ is defined where $a_n = f'(n)$.

(**d**) Determine if the series $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges or diverges. Explain your reasoning.

(**e**) Consider the series $\displaystyle\sum_{n=1}^{\infty} b_n$ where $(1 + 2^{a_n})b_{n+1} = (a_n)!\, b_n$. Use the ratio test to determine if the series $\displaystyle\sum_{n=1}^{\infty} b_n$ converges or diverges.

Free Response Questions Stem Types: Tabular

2020 FRQ Practice Problem BC2

x	1	2	4	5	8
$f(x)$	-1	4	0	1	7
$f'(x)$	-6	0	1	2	4
$g(x)$	5	4	10	12	16
$g'(x)$	-4	5	4	3	1

BC2: The functions f and g are twice differentiable for all values of x. Selected values of f, g and their derivatives f' and g' are given in the table above.

(**a**) Let $y = Q(t)$ be the particular solution to the logistic differential equation $\frac{dy}{dt} = 3y(g(4) - y)$. Find the rate when $Q(t)$ is increasing the fastest.

A large grocery store, customers are entering and exiting the checkout lines. At time $t = 4$ minutes, there are 84 people waiting in line so the manager decides to open up several more checkout lanes. For $4 \leq t \leq 8$ minutes, the rate that customers enter a check out line is given by $f'(t)$ and the rate that customers exit a check out line is given by $g'(t)$ where f' and g' are measured in people per minute.

(**b**) Is the number of customers in line increasing or decreasing at time $t = 4$ minutes?

(**c**) Find the number of customers in line at time $t = 8$ minutes.

The problem has been restated.

x	1	2	4	5	8
$f(x)$	-1	4	0	1	7
$f'(x)$	-6	0	1	2	4
$g(x)$	5	4	10	12	16
$g'(x)$	-4	5	4	3	1

(**d**) Let $s(x) = \dfrac{g(x)}{3x}$. Find $s'(2)$.

Let $H(x) = 3x + \displaystyle\int_1^{x^2} g(x)\,dx$.

(**e**) Find $H'(2)$ and $H''(2)$.

(**f**) Find the second degree Taylor polynomial for $H(x)$ centered at $x = -1$.

Free Response Questions Stem Types: Tabular
2020 FRQ Practice Problem BC3

x	1	2	3	4	5
$f'(x)$	62	30	20	15	12

BC3: The function f is twice differentiable for $x \geq 1$ where $f(5) = -6$. Selected values of the positive and decreasing function f', the derivative of f, are given in the table above. The graph of f' has horizontal asymptote $y = 0$.

The series $\sum_{n=1}^{\infty} a_n$ is defined where $a_n = f'(n)$.

(**a**) Evaluate $\int_{-1}^{0} f''(1-3x)dx$.

(**b**) Evaluate $\int_{5}^{\infty} f''(x)\sin\left(f'(x)\right) dx$.

(**c**) Write an equation of the line tangent to $f(x)$ at $x = 5$. Use the tangent line to approximate $f(5.1)$.

The problem has been restated.

x	1	2	3	4	5
$f'(x)$	62	30	20	15	12

BC3: The function f is twice differentiable for $x \geq 1$ where $f(5) = -6$. Selected values of the positive and decreasing function f', the derivative of f, are given in the table above. The graph of f' has horizontal asymptote $y = 0$.

The series $\sum_{n=1}^{\infty} a_n$ is defined where $a_n = f'(n)$.

(**d**) Use Euler's method, starting at $x = 5$ with two steps of equal size, to approximate $f(3)$.

(**e**) Use a left Riemann sum with the four subintervals indicated in the table to approximate the arc length of $f(x)$ over the interval $x = 1$ to $x = 5$.

(**f**) For $x \geq 6, f'(x) = \dfrac{100}{2^x}$. Find $\sum_{n=6}^{\infty} a_n$.

Free Response Questions Stem Types: Tabular

2020 FRQ Practice Problem BC4

x	1	2	4	5	8
$f(x)$	-1	4	0	1	7
$f'(x)$	-6	0	1	2	4
$g(x)$	5	4	10	12	16
$g'(x)$	-4	5	4	3	1

BC4: The functions f and g are twice differentiable for all values of x. Selected values of f, g and their derivatives f' and g' are given in the table above.

(**a**) Let k be the function defined by $k(x) = \begin{cases} f'\big(g(x)\big), & x \leq 1 \\ x + \cos\big(f(x)\big), & x > 1 \end{cases}$. Is k continuous at $x = 1$?

Why or why not?

(**b**) Let $h(x) = f\big(g(2x)\big)$. Find $h'(1)$.

(**c**) Find $\displaystyle\int_2^4 f'(2x-3)\,dx$.

The problem has been restated.

x	1	2	4	5	8
$f(x)$	-1	4	0	1	7
$f'(x)$	-6	0	1	2	4
$g(x)$	5	4	10	12	16
$g'(x)$	-4	5	4	3	1

BC4: The functions f and g are twice differentiable for all values of x. Selected values of f, g and their derivatives f' and g' are given in the table above.

(**d**) Use Euler's method with two steps of equal size starting at $x = 1$ to approximate $f(3)$.

A large grocery store, customers are entering and exiting the checkout lines. At time $t = 4$ minutes, there are 84 people waiting in line so the manager decides to open up several more checkout lanes. For $4 \leq t \leq 8$ minutes, the rate that customers enter a check out line is given by $f'(t)$ and the rate that customers exit a check out line is given by $g'(t)$ where f' and g' are measured in people per minute.

(**e**) Approximate $f''(4.5)$. Using correct units, interpret the meaning of this value in context of the problem.

(**f**) Is there a time t for $4 < t < 8$ such that the number of customers in line is not changing? Give a reason for your answer.

Free Response Questions Stem Types: Tabular

2020 FRQ Practice Problem BC5

x	1	2	4	5	8
$f(x)$	-1	4	0	1	7
$f'(x)$	-6	0	1	2	4
$g(x)$	5	4	10	12	16
$g'(x)$	-4	5	4	3	1

BC5: The functions f and g are twice differentiable for all values of x. Selected values of f, g and their derivatives f' and g' are given in the table above.

(**a**) Let $m(x) = f(x^3)$. Find $m'(2)$.

(**b**) Evaluate $\lim_{x \to 5} \dfrac{g(f(x)) - x}{x^2 - 25}$.

(**c**) Find $\int_1^8 x g''(x)\, dx$.

(**d**) Let $p(x) = g\left(f'(x)\right)$. Use a right Riemann sum with three subintervals indicated in the table to approximate $\int_2^8 p(x) dx$.

The problem has been restated.

x	1	2	4	5	8
$f(x)$	-1	4	0	1	7
$f'(x)$	-6	0	1	2	4
$g(x)$	5	4	10	12	16
$g'(x)$	-4	5	4	3	1

BC5: The functions f and g are twice differentiable for all values of x. Selected values of f, g and their derivatives f' and g' are given in the table above.

For $t \geq 1$, particles P and Q move along the x axis with velocities $f(t)$ and $g(t)$ respectively. At time $t = 1$, particle P is at position $x = 4$ and particle Q is at position $x = -2$.

(**e**) Use a left Riemann sum with the three subintervals indicated in the table to approximate the position of particle P at time $t = 8$.

(**f**) At $t = 1$, are particles P and Q moving toward or away from each other? Explain your reasoning

(**g**) Is particle Q speeding up or slowing down at time $t = 1$? Give a reason for your answer.