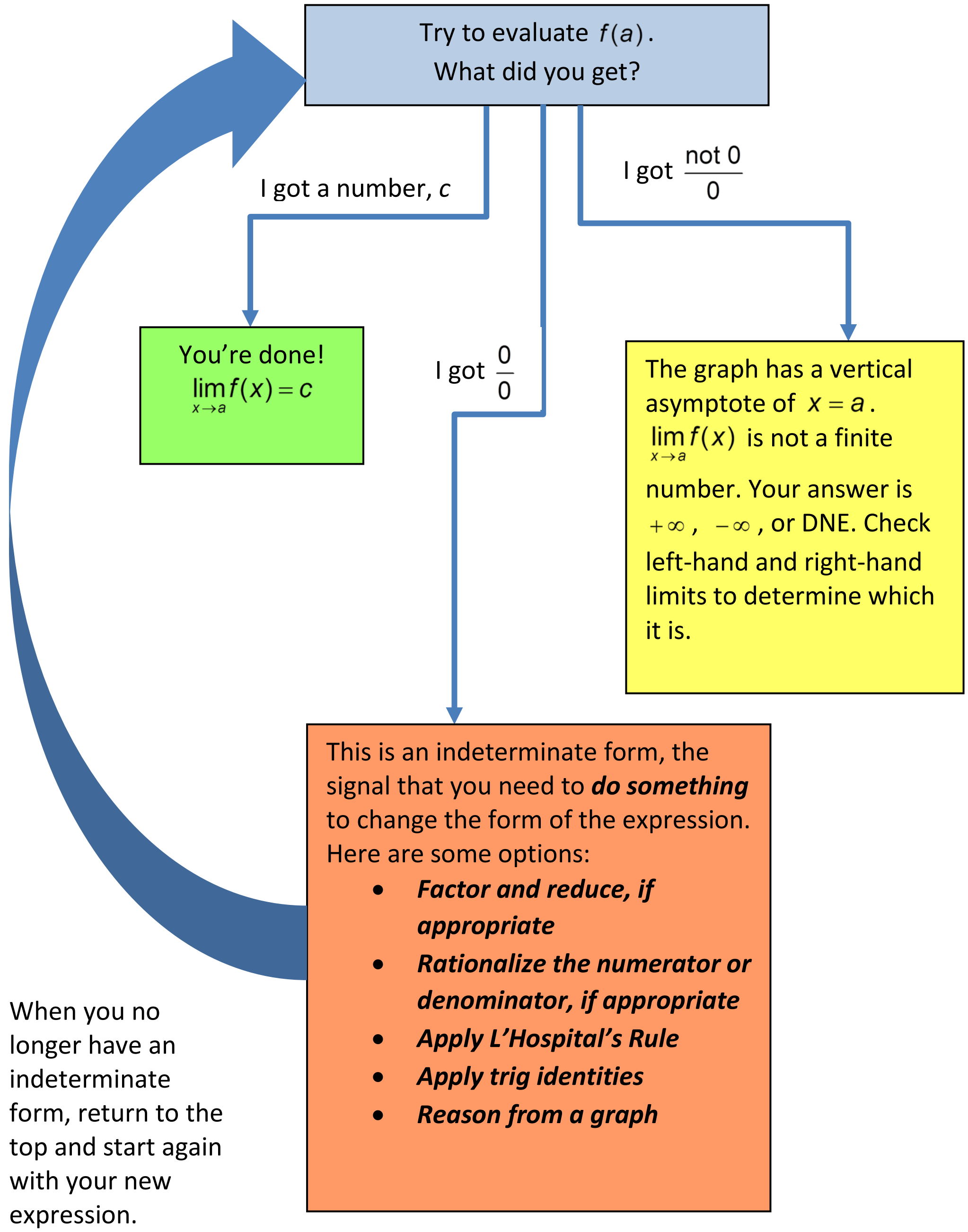
Limit Strategy Flowchart

The following flowchart can help you pick a strategy for evaluating limits of the form $\lim_{x \to a} f(x)$, where $f(x)$ is a rational expression.

Study the flowchart, making sure you understand it. In your textbook, turn to exercises asking you to evaluate a variety of limit expressions, and practice applying the flowchart.



Choosing a Strategy

You have before you a set of cards, each containing a one- or two-sided limit expression of the form $\lim_{x \to a} f(x)$. For each card, evaluate $f(a)$ by plugging the value of a into the given function. Then sort the cards into three piles as follows:

Pile 1: $f(a)$ is a finite number.

Pile 2: As x gets closer and closer to a, the values of $f(x)$ become unbounded.

Pile 3: Plugging a into $f(x)$ results in an indeterminate form of $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

Once your group has completed the sorting, have your teacher check your work before proceeding. Then, working one pile at a time, follow the guidelines below to write each limit expression and its answer in the space provided.

Pile 1: The answer to the given limit expression is $f(a)$.

Pile 2: As x approaches a, $f(x)$ is unbounded. So the answer to $\lim_{x \to a} f(x)$ is either $+\infty, -\infty$, or DNE.

Pile 2 (continued):

Pile 3: This is an indeterminate form. In order to determine the answer, you must first do something to change the form of the limit expression! Some of the options are to:

- Factor and reduce if the function involves a rational expression
- Rationalize the numerator or denominator if radicals are present
- Use trig identities to rewrite any trig functions
- Look at a graph of the function
- Apply L'Hospital's Rule for indeterminate forms such as $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

In some cases, more than one option may apply; you are free to apply whichever option you choose. Also note that once an option has been applied, the resulting expression may now be reclassified as Pile 1, Pile 2, or Pile 3. In other words, you may still have some work to do to determine the answer to the limit statement.

Pile 3 (continued):

Independent Practice: Apply Your Understanding

For each of the following limit expressions of the form $\lim_{x \to a} f(x)$, first determine whether the limit follows from continuity (that is, $f(a)$ is finite, whether the function becomes unbounded as x approaches a, or whether an indeterminate form results. Then choose an appropriate strategy if necessary and determine the limit. Show your work and explain your reasoning clearly.

For each of the following limit expressions of the form $\lim_{x \to \infty} f(x)$, determine the limit by considering what happens to each part of the function expression as x becomes increasingly large.

1. $\lim_{x \to 3} \frac{x^2 + x - 12}{x^2 - 2x - 3}$	2. $\lim_{x \to -1} \frac{x^2 - 4}{x^2 + x - 12}$
3. $\lim_{x \to 4^+} \frac{2 - x}{x - 4}$	4. $\lim_{x \to \infty} \frac{x + 3}{x^2 + 9}$
5. $\lim_{x \to 2^-} g(x)$, where $g(x) = \begin{cases} 3x - 7, & \text{for } x < 2 \\ 5, & \text{for } x = 2 \\ 1 - x^2, & \text{for } x > 2 \end{cases}$	6. $\lim_{x \to \infty} \frac{2}{3 + e^{-x}}$

7. $\lim_{x \to -2} \frac{\sqrt{x+6}-2}{x+2}$	8. $\lim_{h \to 0} \frac{\sqrt{h+9}-3}{h}$
9. $\lim_{y \to \infty} \frac{\ln y}{y-1}$	10. $\lim_{x \to 0} \sin\frac{1}{x}$
11. $\lim_{u \to \infty} \frac{u^2}{e^u - u - 1}$	12. $\lim_{x \to 0^+} \frac{\cos(\pi x)-1}{\ln x}$