Extra Practice Answers

1. $\sum_{n=0}^{\infty} (-1)^n \frac{n+2}{n^3+6}$ $\quad a_n = \frac{n+2}{n^3+6} \quad b_n = \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = 1 > 0$ and $\sum b_n$ converges (p-series, $p>1$)

So $\sum a_n$ converges by LCT.

Therefore $\sum_{n=0}^{\infty} (-1)^n \frac{n+2}{n^3+6}$ is $\boxed{\text{Absolutely convergent.}}$

2. $\sum_{n=1}^{\infty} \frac{\sin n}{n^3+1}$ $\quad a_n = \frac{|\sin n|}{n^3+1} \le \frac{1}{n^3+1} \le \frac{1}{n^3} = b_n$

Positive Term Series. (AC = C)

Since $\sum b_n$ converges (p-series: $p>1$) and $a_n \le b_n$ for all $n \ge 1$, Then $\sum a_n$ converges by BCT. So $\sum_{n=1}^{\infty} \frac{\sin n}{n^3+1}$ is $\boxed{\text{Absolutely Convergent}}$

3. $\sum_{n=0}^{\infty} \frac{n^2}{\sqrt{n^5+7}}$ $\quad a_n = \frac{n^2}{\sqrt{n^5+7}} \quad b_n = \frac{1}{n^{1/2}}$

Positive Term Series diverges

$\lim_{n\to\infty} \frac{a_n}{b_n} = 1 > 0$ so since $\sum b_n$ diverges (p-series: $p<1$), Then $\sum a_n$ $\boxed{\text{diverges by LCT.}}$

4. $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{n^3-3n+4}\right)^4$ $\quad a_n = \left(\frac{n^2+1}{n^3-3n+4}\right)^4 \quad b_n = \frac{1}{n^4}$

Positive Term Series

$\lim_{n\to\infty} \frac{a_n}{b_n} = 1 > 0$ and $\sum b_n$ converges (p-series: $p>1$), so $\sum a_n$ $\boxed{\text{converges by LCT}}$.

5. $\sum_{n=0}^{\infty} (-1)^n \sqrt{\frac{14}{n+15}}$ $\quad a_n = \sqrt{\frac{14}{n+15}} \quad b_n = \frac{1}{\sqrt{n}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \sqrt{14} > 0$ and $\sum b_n$ diverges since p-series ($p<1$), so $\sum a_n$ diverges by LCT. Not Absolutely convergent so $\longrightarrow$

5. continued.

check AST: $\lim_{n\to\infty} a_n = 0$ ✓

$$f(x)=\sqrt{\frac{14}{x+5}} \Rightarrow f'(x)=\frac{1}{2\sqrt{\frac{14}{x+5}}}\cdot\left(\frac{-14}{(x+5)^2}\right)<0$$

for all $x \le 0$, so $f(x)$ is decreasing, Thus $f(n)$ is decreasing for all $n \ge 0$. ✓

So alternating series converges by AST and series is $\boxed{\text{conditionally convergent}}$

6. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$ $\quad f(x)=\frac{1}{x\sqrt{\ln x}}$ $\quad \displaystyle\int_2^{\infty} \frac{1}{x\sqrt{\ln x}}\,dx$

$f(x) > 0$ and continuous ✓ for $x \ge 2$

$$f'(x) = \frac{-\left(\sqrt{\ln x} + x\cdot\frac{1}{2x\sqrt{\ln x}}\right)}{\left(x\sqrt{\ln x}\right)^2}$$

$f'(x) < 0$ for all $x \ge 2$ ✓

$$= \lim_{t\to\infty}\int_2^t \frac{1}{x\sqrt{\ln x}}\,dx$$

$$= \lim_{t\to\infty} 2\sqrt{\ln x}\,\Big|_2^t$$

$$= \lim_{t\to\infty}\left(2\sqrt{\ln t} - 2\sqrt{\ln 2}\right) = \infty$$

So by integral test This positive term series diverges

7. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^{4/3}}$ $\quad f(x)=\frac{1}{x(\ln x)^{4/3}}$ $\quad \displaystyle\int_2^{\infty}\frac{1}{x(\ln x)^{4/3}}\,dx$

$f(x) > 0$ and cont. ✓ for $x \ge 2$

$$f'(x) = \frac{-\left((\ln x)^{4/3} + x\cdot\frac{4}{3}(\ln x)^{1/3}\cdot\frac{1}{x}\right)}{\left[x(\ln x)^{4/3}\right]^2} < 0$$

for $x \ge 2$ ✓

$$= \lim_{t\to\infty}\int_2^t \frac{1}{x(\ln x)^{4/3}}\,dx$$

$$= \lim_{t\to\infty} \frac{-3}{(\ln x)^{1/3}}\,\Big|_2^t$$

$$= \lim_{t\to\infty}\left(\frac{-3}{(\ln t)^{1/3}} + \frac{3}{(\ln 2)^{1/3}}\right)$$

$$= \frac{3}{\sqrt[3]{\ln 2}}$$ So converges by integral test.

Therefore series is $\boxed{\text{Absolutely Convergent}}$

8. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln(\ln n)}$

Since $n > \ln n$ for all $n \geq 3$,

Then $n > \ln(\ln n)$ also.

Therefore $\frac{1}{n} < \frac{1}{\ln(\ln n)}$ for all $n \geq 3$

$a_n = \frac{1}{\ln(\ln n)}$

Since $\sum \frac{1}{n}$ diverges (harmonic series),

Then $\sum \frac{1}{\ln(\ln n)}$ $\boxed{\text{diverges}}$ by BCT.

9. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \cdot \frac{1}{\ln(\ln n)}$ From above we know $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln(\ln n)}$ diverges.

So not Absolutely convergent.

check AST: $\displaystyle\lim_{n\to\infty} \frac{1}{\ln(\ln n)} = 0$ ✓

let $f(x) = \frac{1}{\ln(\ln x)}$

$f'(x) = -\frac{1}{x \ln x (\ln(\ln x))^2} < 0$ for $x \geq 3$

So since $f(x)$ is decreasing, $f(n)$ is also.

Therefore by AST, $\sum (-1)^n \frac{1}{\ln(\ln n)}$ converges.

So $\boxed{\text{Conditionally Convergent}}$

10. $\displaystyle\sum_{n=0}^{\infty} \frac{n^3}{4^n}$ $\displaystyle\lim_{n\to\infty} \frac{(n+1)^3}{4^{n+1}} \cdot \frac{4^n}{n^3} = \lim_{n\to\infty} \frac{(n+1)^3}{4n^3} = \frac{1}{4} < 1$

So $\sum \frac{n^3}{4^n}$ $\boxed{\text{converges by Ratio Test.}}$

11. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-10}{n}\right)^n$ let $a_n = \left(\frac{10}{n}\right)^n$

$\displaystyle\lim_{n\to\infty} \sqrt[n]{\left(\frac{10}{n}\right)^n} = \lim_{n\to\infty} \frac{10}{n} = 0 < 1$

So $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-10}{n}\right)^n$ converges by Root Test

So $\boxed{\text{Absolutely Convergent}}$

12. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{-n}{10}\right)^n$ let $a_n = \left(\frac{n}{10}\right)^n$ $\displaystyle\lim_{n\to\infty} \left(\frac{n}{10}\right)^n = \infty^\infty = \infty$

So $\displaystyle\sum_{n=0}^{\infty} \left(\frac{-n}{10}\right)^n$ $\boxed{\text{diverges}}$ by n^{th} term Test.

13. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n!}{n^n}$ $\qquad \displaystyle\lim_{n\to\infty}\frac{(n+1)!}{(n+1)^{n+1}}\cdot\frac{n^n}{n!}$

$$= \lim_{n\to\infty}\frac{\cancel{(n+1)}\cancel{n!}}{\cancel{(n+1)}(n+1)^n}\cdot\frac{n^n}{\cancel{n!}} = \lim_{n\to\infty}\frac{n^n}{(n+1)^n}$$

$$= \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty}\left(\frac{1}{\frac{n+1}{n}}\right)^n$$

$$= \lim_{n\to\infty}\frac{1}{(1+\frac{1}{n})^n} = \frac{1}{e} < 1$$ So converges by Ratio Test.

Thus $\boxed{\text{Absolutely Convergent}}$

14. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(3^n)}{n^{1.3}}$ $\qquad$ let $a_n = \dfrac{|\cos 3^n|}{n^{1.3}}$

and $b_n = \dfrac{1}{n^{1.3}}$

Since $a_n \le b_n$ for $n \ge 1$ and $\sum b_n$ converges (P-series: $p>1$)

Then $\sum b_n$ converges by BCT.

Thus $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(3^n)}{n^{1.3}}$ is $\boxed{\text{Absolutely Convergant}}$

15. $\displaystyle\sum_{n=1}^{\infty}\frac{n^{10}9^n}{10^n}$ $\qquad \displaystyle\lim_{n\to\infty}\frac{(n+1)^{10}9^{n+1}}{10^{n+1}}\cdot\frac{10^n}{9^n\cdot n^{10}}$

$$= \lim_{n\to\infty}\frac{9(n+1)^{10}}{10\,n^{10}} = \frac{9}{10} < 1$$ So

$\displaystyle\sum_{n=1}^{\infty} n^{10}\left(\frac{9}{10}\right)^n$ $\boxed{\text{converges by Ratio test.}}$

16. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}$ $\qquad \displaystyle\lim_{n\to\infty}\frac{1}{(2n+2)!}\cdot\frac{(2n)!}{1} = \lim_{n\to\infty}\frac{\cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}} = 0 < 1$

So converges by Ratio test.

Thus $\boxed{\text{Absolutely Convergent}}$

17. $\displaystyle\sum_{n=4}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n^5-1000}}$ $\qquad a_n = \frac{1}{\sqrt{n^5-1000}}$ $\quad b_n = \frac{1}{\sqrt{n^5}}$

$\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 1 > 0$ so since

$\sum b_n$ converges (p-series: $p>1$), Then

$\sum a_n$ converges by LCT.

Thus $\boxed{\text{Absolutely Convergent}}$

18. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$ $\qquad a_n = \frac{\ln(n)}{n}$ $\quad b_n = \frac{1}{n}$

Since $\sum \frac{1}{n}$ diverges (harmonic series)

and $\frac{1}{n} \le \frac{\ln n}{n}$ for $n \ge 3$, Then

$\sum \frac{\ln n}{n}$ diverges by BCT.

Check AST: $\displaystyle\lim_{n\to\infty} \frac{\ln n}{n} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$ ✓

let $f(x) = \frac{\ln x}{x}$, $f'(x) = \frac{\frac{1}{x}\cdot x - \ln x}{x^2}$

$= \frac{1-\ln x}{x^2} < 0$ for $x \ge 3$

So $f(n)$ decreasing for $n \ge 3$. ✓

So $\sum (-1) \frac{\ln n}{n}$ converges by AST.

Thus $\boxed{\text{Conditionally Convergent}}$

19. $\displaystyle\sum_{n=1}^{\infty} \sqrt{\arctan n}$ $\qquad \displaystyle\lim_{n\to\infty} \sqrt{\arctan n} = \sqrt{\pi/2} \ne 0$ so

$\displaystyle\sum_{n=1}^{\infty} \sqrt{\arctan n}$ $\boxed{\text{diverges}}$ by n^{Th} Term Test

20. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\arctan n}$ $\qquad \displaystyle\lim_{n\to\infty} \frac{1}{\arctan n} = \frac{2}{\pi} \ne 0$ so

$\sum \frac{(-1)^n}{\arctan n}$ $\boxed{\text{diverges}}$

21. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n}{3-4n}\right)^n$ $\qquad \displaystyle\lim_{n\to\infty} \sqrt[n]{\left(\frac{2n}{4n-3}\right)^n} = \lim_{n\to\infty} \frac{2n}{4n-3} = \frac{1}{2} < 1$

$\displaystyle = \sum_{n=1}^{\infty} (-1)^n \left(\frac{2n}{4n-3}\right)^n$

So $\sum \left(\frac{2n}{4n-3}\right)^n$ converges by Root Test.

Thus $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n}{3-4n}\right)^n$ is $\boxed{\text{Absolutely Convergent}}$

22. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+1}{n+2}$

$\lim_{n\to\infty} \frac{n+1}{n+2} = 1 \neq 0$

So Series $\boxed{\text{diverges by } n^{\text{th}} \text{ term Test.}}$

23. $\sum_{n=1}^{\infty} \frac{1}{1+e^n}$

$a_n = \frac{1}{1+e^n} \quad b_n = \frac{1}{e^n} = \left(\frac{1}{e}\right)^n$

Since $\sum \left(\frac{1}{e}\right)^n$ converges (geo: $|r|<1$)

and $a_n \leq b_n$ for all $n \geq 1$,

Then $\sum \frac{1}{1+e^n}$ $\boxed{\text{converges by BCT.}}$

24. $\sum_{n=1}^{\infty} \frac{1}{1-e^{-n}}$

$\lim_{n\to\infty} \frac{1}{1-\frac{1}{e^n}} = 1 \neq 0$ So $\boxed{\text{diverges}}$ by n^{th} term Test

25. $\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$

$a_n = \frac{1}{n^2 \ln n} \quad b_n = \frac{1}{n^2}$

Since $\sum \frac{1}{n^2}$ converges (p-series: $p>1$)

and $\frac{1}{n^2 \ln n} \leq \frac{1}{n^2}$ for $n \geq 3$,

Then $\sum \frac{1}{n^2 \ln n}$ $\boxed{\text{converges by BCT.}}$