HATT Ch R-2 Practice

Name: Key   Date: Seat:

1. (3 points) If universal set
   $U = \{0, 1, 2, 3, 4, 5, 6, 7\}$, and
   $A = \{1, 2, 3, 5\}$,
   $B = \{2, 3, 5, 7\}$,
   $C = \{1, 4, 6\}$,
   find $(A \cup B) \cap \overline{C}$.

$A \cup B = \{1, 2, 3, 5, 7\}$

$\overline{C} = \{2, 3, 5, 7\}$

$(A \cup B) \cap \overline{C} = \{2, 3, 5, 7\}$

2. (4 points) Determine the domain of the variable in the expression, stating your answer in interval notation.

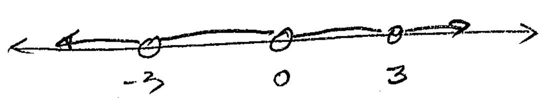
$$\frac{x^2 + 7x + 12}{x^3 - 9x}$$

Avoid division by zero:

$x^3 - 9x \neq 0$

$x(x-3)(x+3) \neq 0$

Avoid $0, \pm 3$, Domain:



$(-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty)$

3. (4 points) Use synthetic division to find the quotient and the remainder:

$$x^3 + 3x^2 - 4 \text{ divided by } x + 2$$

$$\begin{array}{r|rrrr} -2 & 1 & 3 & 0 & -4 \\ & & -2 & -2 & 4 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

quotient:

$x^2 + x - 2$

remainder:

0

4. (6 points) Solve for x.

$$\frac{x^2-3x+4}{x^3-1} = \frac{5}{x^2+x+1} + \frac{2}{x^3-1}, x \neq 1$$

$$\frac{x^2-3x+2}{(x-1)(x^2+x+1)} = \frac{5}{x^2+x+1}$$

$$\frac{(x-1)(x-2)}{(x-1)(x^2+x+1)} = \frac{5}{x^2+x+1}$$

$$x-2 = 5$$

$$x = 7$$

5. (5 points) Solve for t

$$t^{1/2} - 2t^{1/4} = -1$$

Let $u = t^{1/4}$

$$u^2 - 2u + 1 = 0$$

$$(u-1)(u-1) = 0$$

$$u = 1$$

$$t^{1/4} = 1$$

$$t = 1^4 = 1$$

6. (3 points) In order to complete the square, what should be added to the equation (DO NOT Solve for x) ?

$$x^2 + \frac{1}{3}x = 3$$

the co-eff of x is $\frac{1}{3}$

so add

$$\left(\frac{1}{2\cdot 3}\right)^2 = \frac{1}{36}$$

to both sides

7. (4 points) Solve for x

$$\left|\frac{x}{3}+\frac{2}{5}\right| = 2$$

If $\frac{x}{3}+\frac{2}{5}=2$

then $5x+6=30$

$x=\frac{30-6}{5}$

$x=\frac{24}{5}$

If $\frac{x}{3}+\frac{2}{5}=-2$

$5x+6=-30$

$x=\frac{-30-6}{5}$

$x=-\frac{36}{5}$

$x\in\left\{-\frac{36}{5},\frac{24}{5}\right\}$

8. (3 points) Find the x and y intercepts and label which is which. If none, state so:

$$4x^2+y^2=4$$

If $y=0$ $4x^2=4$

$x=\pm 1$

If $x=0$

$y^2=4$

$y=\pm 2$

Hence the intercepts are

$(-1,0),(1,0),$

$(0,-2),(0,2)$

9. (5 points) Solve for x, stating your answer in interval notation.

$$-3 \leq \frac{3x-4}{2} \leq 6$$

$-6\leq 3x-4\leq 12$

$-10\leq 3x\leq 16$

$\frac{-10}{3}\leq x\leq\frac{16}{3}$

In interval notation:

$\left[\frac{-10}{3},\frac{16}{3}\right]$

10. (5 points) Solve the inequality, using the conditional number line and stating your answer in interval notation

$$x^2 - 2x < 8$$

$x^2 - 2x - 8 < 0$

$(x+2)(x-4) < 0$

$(-2, 4)$

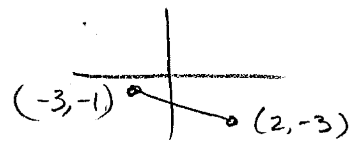
11. (4 points) Simplify

$$\frac{(-2)^3 x^4 (yz)^2}{3^2 x y^3 z}$$

$\dfrac{-8x^3 z}{9y}$

12. (4 points) Write the equation of the line containing the points

$$f(-3) = -1$$
$$f(2) = -3$$



$m = \dfrac{-2}{5}$

$y + 1 = \frac{-2}{5}(x+3)$

or $5y + 5 = -2x - 6$

or $2x + 5y = -11$

or

$y = \frac{-2}{5}x - \frac{11}{5}$

etc....

13. (5 points) Write the equation of the line perpendicular to the line $3x - y = -4$ and containing the point $(-2, 4)$.

Find slope of $3x - y = -4$:

$y = 3x - 4$

slope is 3

the $\perp$ slope is $-\frac{1}{3}$

$y - 4 = -\frac{1}{3}(x+2)$

14. (4 points) Simplify

$$\frac{\frac{x-2}{x+2}+\frac{x-1}{x+1}}{\frac{x}{x+1}-\frac{2x-3}{x}}$$

$$\left(\frac{(\cancel{x+1})(x-2)+(x-1)(\cancel{x+1})}{(x+2)(\cancel{x+1})}\right)\cdot\frac{x(x+1)}{x^2-(2x-3)(x+1)}$$

$2x^2-x-3$

$$\frac{2x-3}{x+2}\cdot\frac{x(x+1)}{-x^2+x+3}$$

$x\ \ -x-3$

$$\frac{x^2+3x-3}{(x+2)(-x^2+x+3)}$$

15. (4 points) Convert the following complex expressions into standard $a+bi$ form, showing more than one step.

(a) $\frac{3}{(3+i)}\frac{(3-i)}{(3-i)}$

$$\frac{9-3i}{9+1}$$

$$\frac{9}{10}-\frac{3}{10}i$$

(b) i^{153}

$(i^4)^{38}(i)=i$

16. (3 points) Rationalize the denominator

$$\frac{\sqrt{3}}{(1+3\sqrt{2})}\frac{(1-3\sqrt{2})}{(1-3\sqrt{2})}$$

$$\frac{\sqrt{3}-3\sqrt{6}}{1-18}$$

$$\frac{\sqrt{3}-3\sqrt{6}}{-17}$$

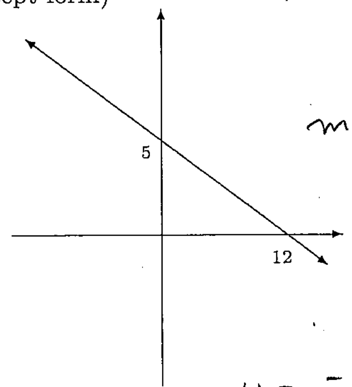
17. (3 points) Multiply $(2x-5)(x^2+x+1)$

$$\begin{array}{r} 2x^3 + 2x^2 + 2x \phantom{{}-5} \\ + \quad -5x^2 - 5x - 5 \\ \hline 2x^3 - 3x^2 - 3x - 5 \end{array}$$

18. (4 points) Write the general formula to describe the variation: The square of T varies directly with the cube of a and inversely with the square of d.

$$T^2 = \frac{k\,a^3}{d^2}$$

19. (4 points) What is the equation of the line in the graph below? (use either general or slope intercept-form)



$m = \frac{-5}{12}$

$y = \frac{-5}{12}x + 5$

or $5x + 12y = 60$

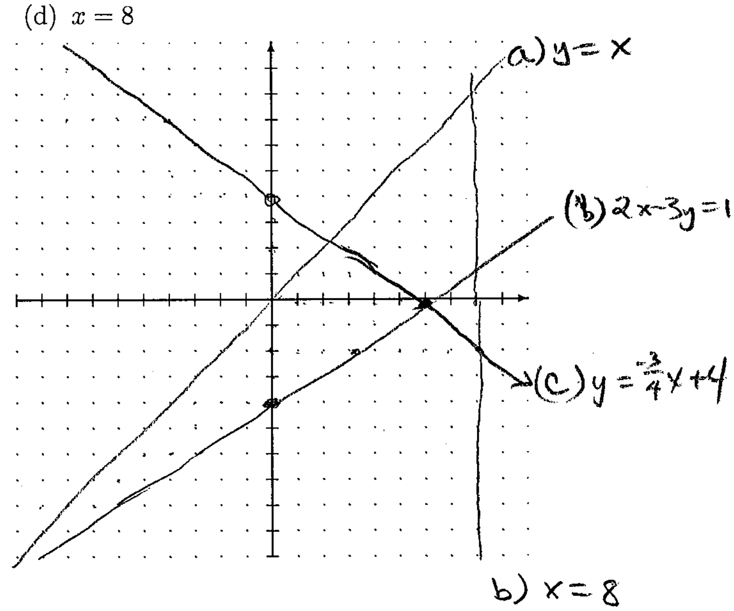
20. (8 points) Graph the following equations on the axis below and label the lines a, b, c and d

(a) $y = x$

(b) $2x - 3y = 12$

(c) $y = -\frac{3}{4}x + 4$

(d) $x = 8$



25. (8 points) The volume V of an ideal gas varies directly with the temperature T and inversely with the pressure P. If 100 liters of oxygen at a temperature of 300K has a pressure of 24 atmospheres, find the pressure of the oxygen when the temperature is increased to 310K and the volume was reduced to only 80 liters.

(a) Establish variables or label a diagram.

k = constant of proportionality

(V, T, P already established in the question,

(b) Write an equation(s) using these variables

$$V = \frac{kT}{P}$$

$$k = \frac{VP}{T} = \frac{(100)(24)}{300} = 8 \;; \quad V = \frac{8T}{P}$$

(c) Solve the equation(s) showing all steps.

$$V = \frac{8T}{P}$$

$$P = \frac{8T}{V} = \frac{8(310)}{80}$$

$$P = 31$$

(d) Answer the question in a complete sentence.

The pressure would be 31 atmospheres.

21. (4 points) What is the coordinate of the point of intersection of line $3x + 2y = 4$ with the line $4x - 5y = -33$?

$$\begin{cases} 3x + 2y = 4 \\ 4x - 5y = -33 \end{cases}$$

$$\begin{cases} 15x + 10y = 20 \\ 8x - 10y = -66 \end{cases}$$

$$23x = -46$$

$$x = -2$$

when $x = -2$

$$-6 + 2y = 4$$

$$y = 5$$

Hence the intersection is $(-2, 5)$

22. (2 points) What is the midpoint between $(5h, -3\pi)$ and $(7h, -7\pi)$?

$$\left(\frac{5h+7h}{2}, \frac{-3\pi - 7\pi}{2}\right)$$

$$(6h, -5\pi)$$

23. (6 points) What is the center and radius of the circle

$$x^2 + y^2 + 4x - 4y - 1 = 0$$

$$(x^2 + 4x + 4) + (y^2 - 4y + 4) = 1 + 4 + 4$$

$$(x+2)^2 + (y-2)^2 = 9$$

center $(-2, 2)$

radius 3

24. (4 points) What is the general form of the equation of a circle that is centered at $(-3, 2)$ with radius 5?

$$(x+3)^2 + (y-2)^2 = 25$$

$$x^2 + y^2 + 6x - 4y - 12 = 0$$

26. (8 points) A boat can travel at 30 miles per hour in still water. It carries enough fuel for 2 hours. The current the river is 5 miles per hour, so if it travels downstream, how far can it travel before it turns around so that it returns to its original starting position just as it runs out of fuel?

(a) Establish variables or label a diagram.

	rate	time	dist
downstream	30 + 5	t	35t
upstream	30 − 5	5 − t	125 − 25t

t = time going downstream

x = distance before returning

(b) Write an equation(s) using these variables

$$35t = 125 - 25t$$

$$t = \frac{125}{60} = \frac{25}{12} \text{ hours}$$

$$x = 35\left(\frac{25}{12}\right) = \frac{\quad}{12}$$

(c) Solve the equation(s) showing all steps.

$$35t = 125 - 25t$$

$$60t = 125$$

$$t = \frac{25}{12} \text{ hours}$$

$$x = 35\left(\frac{25}{12}\right) = \frac{875}{12} = 72.91\bar{6} \text{ miles}$$

(d) Answer the question in a complete sentence.

The boat can travel $72\frac{11}{12}$ miles downstream before it turns arond so that it returns to its original position without running out of fuel.

Table 1: For office use only

Page	score	max	topics
1		11	sets, domain, synthetic division
2		14	factoring difference of cubes, quadratic form of fractional exponents, abstract understanding of the completing the square technique
3		12	absolute value, intercepts, linear inequality
4		18	Quadratic inequality, negative exponents, line equation from points, functional notation, perpendicular slope, point-slope form of a line
5		11	complex fractions, complex numbers, rationalizing denominators
6		19	multiplying polynomials, variation, graphing lines
7		16	solving for x and y (substitution or simultaneous), midpoint formula, equations of circles, application of completing the square technique
8		8	variation word problem
9		8	basic word problem (non-trivial problem solving)
Total		117	