HATT Practice Test 13

Name: Seat:

Some Sum Formulas

Arithmetic sum of first n terms:

$$\sum_{k=1}^{n} a_n = \frac{n(a_1 + a_n)}{2}$$

Geometric sum of first n terms (where $r = \frac{a_{n+1}}{a_n} \neq 0$):

$$\sum_{k=1}^{n} a_1 r^{n-1} = a_1 \left(\frac{1 - r^n}{1 - r} \right)$$

Geometric Sum of ∞ terms if $|r| < 1$

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a_1}{1 - r}$$

Binomial Expansion of $(a + b)^n$

$$(a + b)^n = \sum_{k=0}^{n} {}_nC_k a^{n-k} b^k$$

1. (3 points) Consider $\{5, 2, \frac{4}{5}, \frac{8}{25}, \frac{16}{125}, \frac{32}{625}, \dots\}$
   (a) What is the n^{th} term?

   (b) What is the sum of the first 13 terms?

   (c) If possible, determine the sum of all of the terms

2. (3 points) Consider $\{3, 15, 75, 375, 1875, \dots\}$
   (a) Is this sequence arithmetic or geometric or neither?

   (b) What is the nth term?

   (c) What is the sum of the first 10 terms?

3. Write out the first three terms of $\left\{ \frac{n-1}{2n} \right\}$

4. A recursive sequence is defined $a_1 = 3$, $a_n = 2na_{n-1}$. Write the first three terms.

5. (3 points) Consider $\{a_n\} = \{7, 3, -1, -5, -9, \dots\}$

(a) Is this sequence arithmetic, geometric or neither?

(b) What is the n^{th} term?

(c)

$$\sum_{k=1}^{19} a_k =$$

6. Expand$(2a + 3)^4$

7. Compute

$$\binom{12}{7}$$

8. Compute

$$\sum_{k=1}^{300} 4 - 2k$$

9. Compute the infinite sum

$$18 + \frac{21}{2} + \frac{49}{8} + \frac{343}{96} + \frac{2401}{1152} + \cdots$$

10. What is the x^5 term of $(3x + 2)^{13}$?

11. Show how to compute

$$\sum_{k=1}^{60} 30k - 2$$

12. Solve for x:

$$\binom{x+1}{x-1} = 3003$$

13. Prove by induction that

$$\sum_{k=1}^{n} 6k - 3 = 3n^2$$

14. Prove by induction that

$$8 + 16 + \cdots + 8(2^{n-1}) = 2^{n+3} - 8$$

15. The first two rows of seats in a amphitheater have 8 seats each. The next two rows have 10 seats each. The next two rows have 12 seats each, and so on until the last two rows have 40 seats each.

    (a) How many rows are there in the amphitheater?

    (b) How many seats are in the amphitheater?

16. In a race, Achilles allows the tortoise a 500 pous head start. When Achilles reaches the place where the tortoise started the tortoise has moved another 400 podes. When Achilles reaches that spot, the tortoise has moved another 320 podes. When Achilles reaches that spot, the tortoise has moved another 256 podes. How many podes will Achilles be from his start line when he reaches the tortoise (i.e., when this happens infinitely many times, how far has Achilles traveled)?